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Departmentof

ELECTRICAL & ELECTRONICS ENGINEERING

ELECTRICAL CIRCUIT ANALYSIS-II

SUBJECT MATERIAL

YEAR :II SEMESTER :I

Regulaton:VR23 **SubjectCode:VR2321202**

Prepared by

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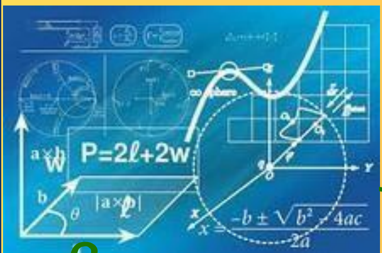
EEE Department



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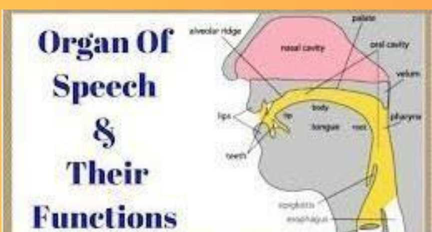

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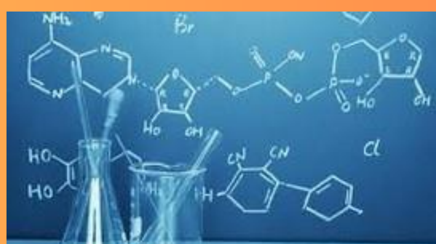

ELECTRICAL CIRCUIT ANALYSIS-II

II B.TECH II SEM

FOR

EEE

(AUTONOMOUS)

(R23)

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

V S M COLLEGE OF ENGINEERING

RAMCHANDRAPURAM

E.G. Dt. - 533255

II Year –I SEMESTER

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ELECTRICAL CIRCUIT ANALYSIS-II

Pre-requisite: Analysis of DC and Single phase AC Circuits, Concepts of differentiation and integration.

Course Objectives:

- To understand three phase circuits
- To analyse transients in electrical systems
- To evaluate network parameters of given electrical network
- To apply Fourier analysis to electrical systems
- To understand graph theory for circuit analysis and to understand the behaviour of filters

Course Outcomes:

At the end of the course, student will be able to,

CO1: Analyse the balanced and unbalanced 3 phase circuits for power calculations.
CO2: Analyse the transient behaviour of electrical networks in different domains.
CO3: Estimate various Network parameters.
CO4: Apply the concept of Fourier series to electrical systems.
CO5: Analyse the filter circuit for electrical circuits.

UNIT - I

Analysis of three phase balanced circuits:

Phase sequence, star and delta connection of sources and loads, relation between line and phase quantities, analysis of balanced three phase circuits, measurement of active and reactive power.

Analysis of three phase unbalanced circuits:

Loop method, Star-Delta transformation technique, two-wattmeter method for measurement of three phase power.

UNIT – II

Laplace transforms – Definition and Laplace transforms of standard functions– Shifting theorem – Transforms of derivatives and integrals, Inverse Laplace transforms and applications.

Transient Analysis: Transient response of R-L, R-C and R-L-C circuits (Series and parallel combinations) for D.C. and sinusoidal excitations – Initial conditions - Solution using differential equation approach and Laplace transform approach.

UNIT - III

Network Parameters: Impedance parameters, Admittance parameters, Hybrid parameters, Transmission (ABCD) parameters, conversion of Parameters from one form to other, Conditions for Reciprocity and Symmetry, Interconnection of Two Port networks in Series, Parallel and Cascaded configurations- problems.

UNIT - IV

Analysis of Electric Circuits with Periodic Excitation: Fourier series and evaluation of Fourier coefficients, Trigonometric and complex Fourier series for periodic waveforms, Application to Electrical Systems – Effective value and average value of non-sinusoidal periodic waveforms, power factor, effect of harmonics

UNIT - V

Filters: Classification of filters-Low pass, High pass, Band pass and Band Elimination filters, Constant-k filters -Low pass and High Pass, Design of Filters.

Textbooks:

1. Engineering Circuit Analysis, William Hayt and Jack E. Kemmerly, 8th Edition McGraw-Hill, 2013
2. Fundamentals of Electric Circuits, Charles K. Alexander, Mathew N. O. Sadiku, 3rd Edition, Tata McGraw-Hill, 2019

Reference Books:

1. Network Analysis, M. E. Van Valkenburg, 3rd Edition, PHI, 2019.
2. Network Theory, N. C. Jagan and C. Lakshminarayana, 1st Edition, B. S. Publications, 2012.
3. Circuits and Networks Analysis and Synthesis, A. Sudhakar, Shyam Mohan S. Palli, 5th Edition, Tata McGraw-Hill, 2017.
4. Engineering Network Analysis and Filter Design (Including Synthesis of One Port Networks)- Durgesh C. Kulshreshtha Gopal G. Bhise, Prem R. Chadha ,Umesh Publications 2012.
5. Circuit Theory: Analysis and Synthesis, A. Chakrabarti, Dhanpat Rai & Co., 2018, 7th Revised Edition.

Online Learning Resources:

1. https://archive.nptel.ac.in/courses/117/106/117106108/
2. https://archive.nptel.ac.in/courses/108/105/108105159/

UNIT-I

Balanced and unbalanced three phase circuits

Analysis of three phase balanced circuits:-

phase sequence, star and delta connection of source & loads, relation betweens line and phas voltage and currents, analysis of balanced three phase circuits, measurement of active and reactive power

Analysis of three phase unbalanced circuits

loop method, star-delta transformation technique, two-wattmeter method for measurement of three phase power.

UNIT-II

Transient response of first order (R-L, R-C) & second order (R-L-C) circuit using differential equations

Transient response of first order (R-L, R-C) & second order (R-L-C) using lapplace transforms

UNIT-III

Transient response of first order (R-L, R-C) & second orde (R-L-C) circuits using diff eqns

Transient response of first order (R-L), R-C) & second order (R-L-C) circuit using laplace transforms

UNIT-IV

Two port network parameter - Z, Y, ABCD & Hybrid parameters & their relations, cascaded networks

UNIT-V

Need of filters - classification - characterestic impedance - low pass filter, high pass filter, band pass filter, band stop or band elimination filter, m-derived filter, design of filters

UNIT - I

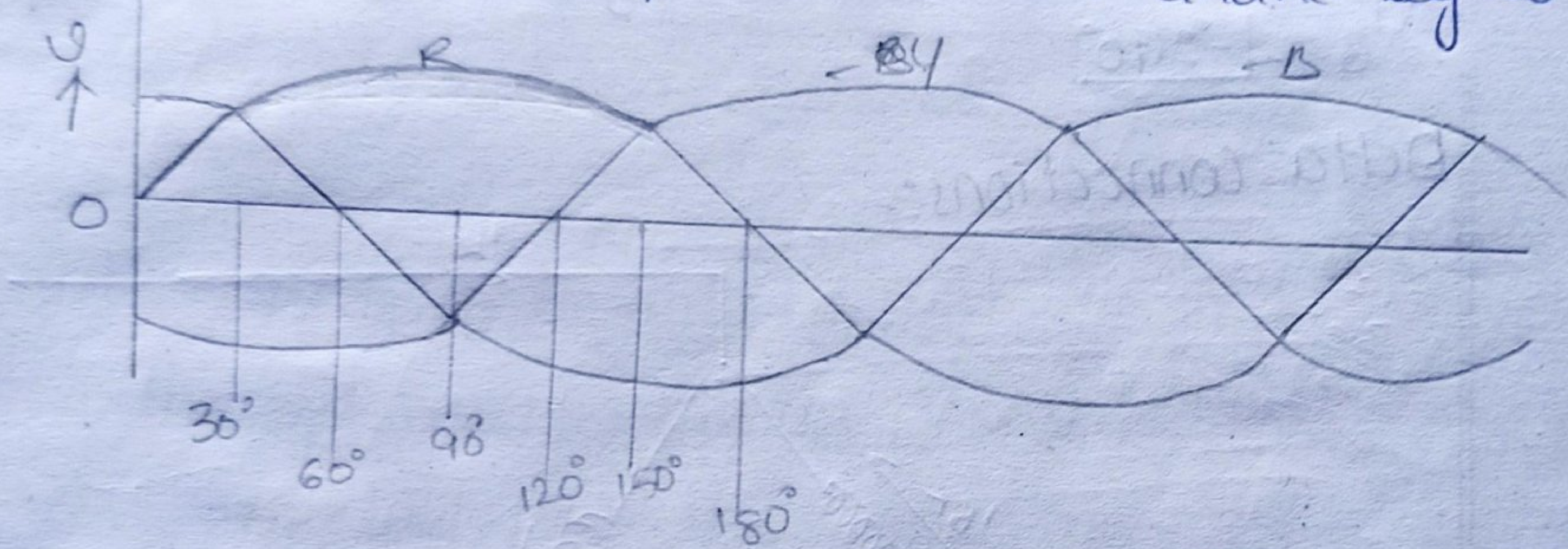
BALANCED AND UNBALACED 3ϕ CIRCUITS

phase sequence:-

The sequence in which the voltages in three phases reach their maximum values is called phase sequence

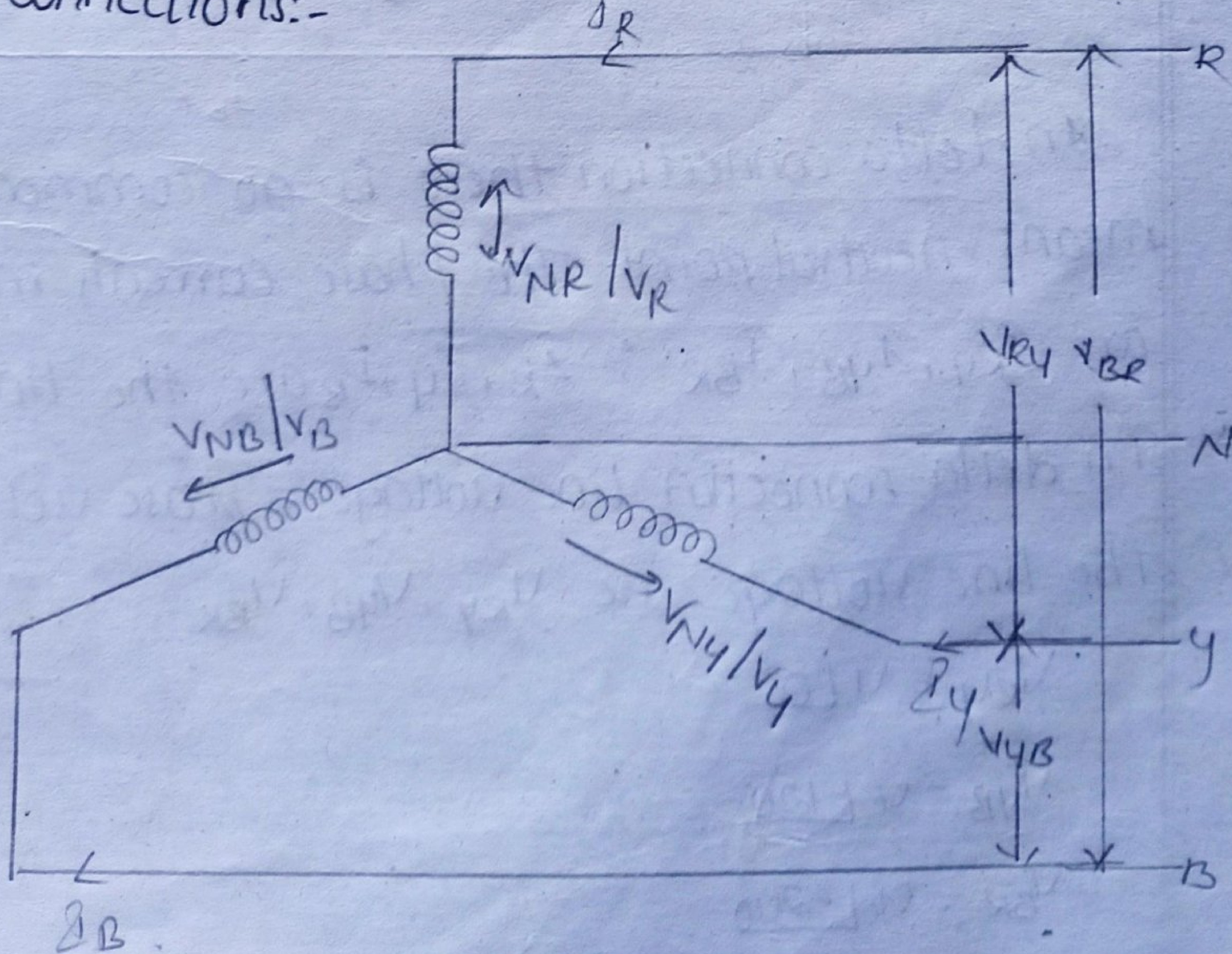
Generally the phase sequence are R, Y, B

Generally the voltages are same magnitude & frequency but are displaced from one another by 120



The 3 phase system are Either (Y) star or delta(Δ) connections

Star connections:-



The star connection, the phases connected in star N be the neutral point.

V_{NR}, V_{NY}, V_{NB} are represented by the phase voltages & simple denoted by V_R, V_Y, V_B

The voltage between lines i.e V_{RY}, V_{YB}, V_{BR} are the voltage &

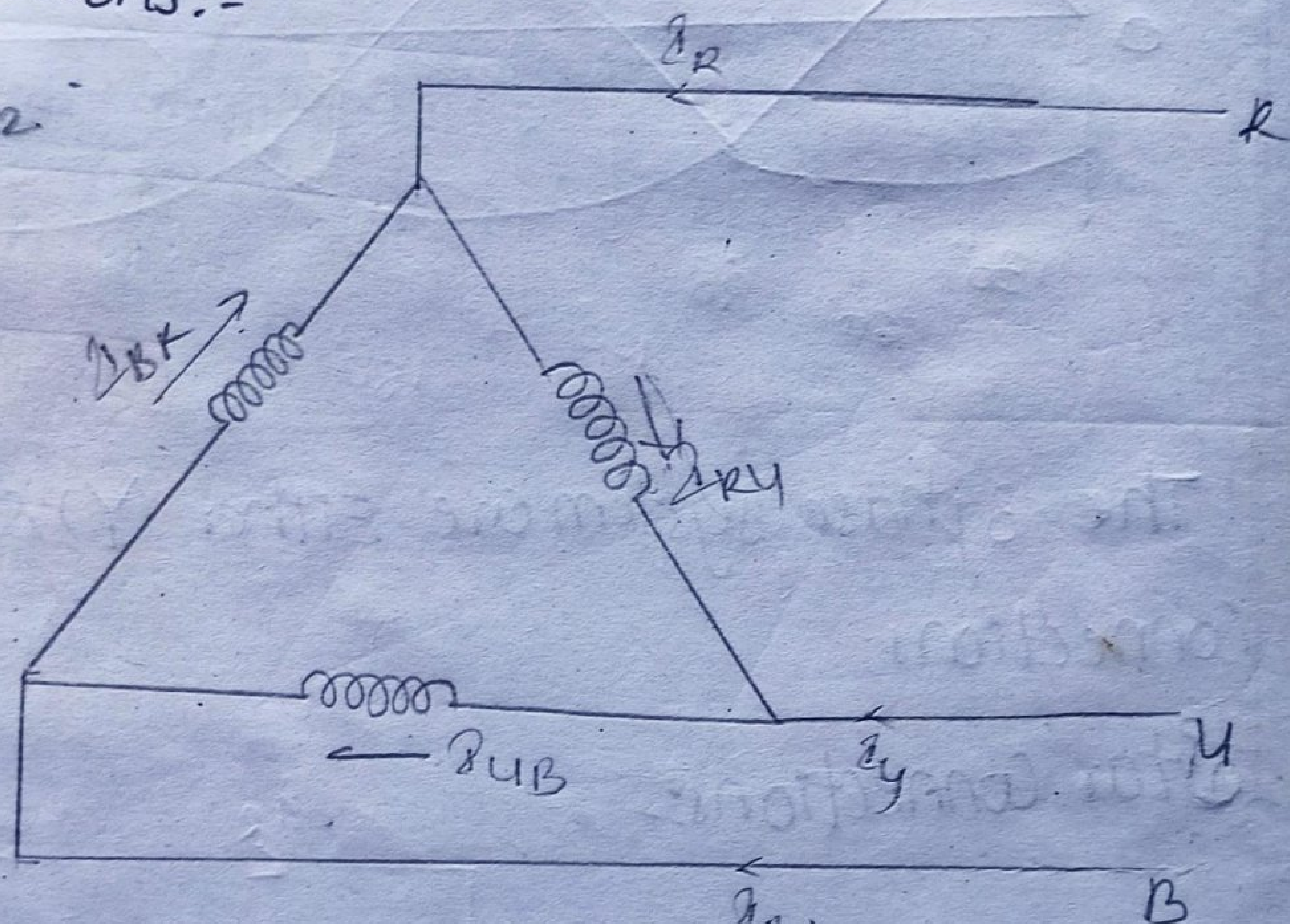
In a balanced 3-phase system $V_R = V\angle 0°$; $V_Y = V\angle{-120°}$

$V_B = \angle{-240°}$

Delta connections:-

120
360

0 -12



In delta connection there is no common point means neutral point. The phase currents in delta are I_{RY}, I_{YB}, I_{BR} ; I_R, I_Y, I_B are the line currents

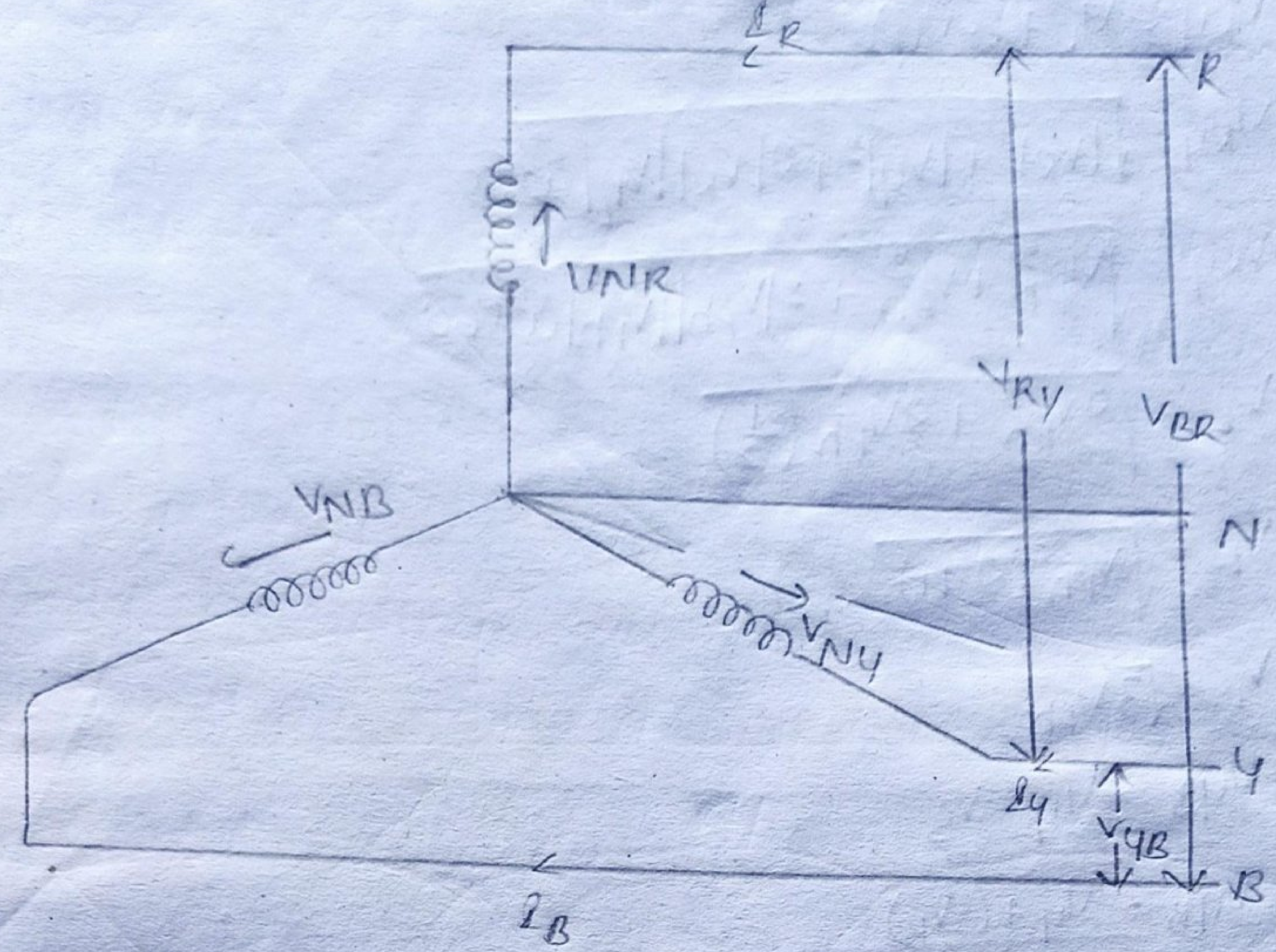
* In delta connection line voltage = phase voltage
* The line voltage are V_{RY}, V_{YB}, V_{BR}.

$V_{RY} = V\angle 0°$

$V_{YB} = V\angle{-120°}$

$V_{BR} = V\angle{-240}$

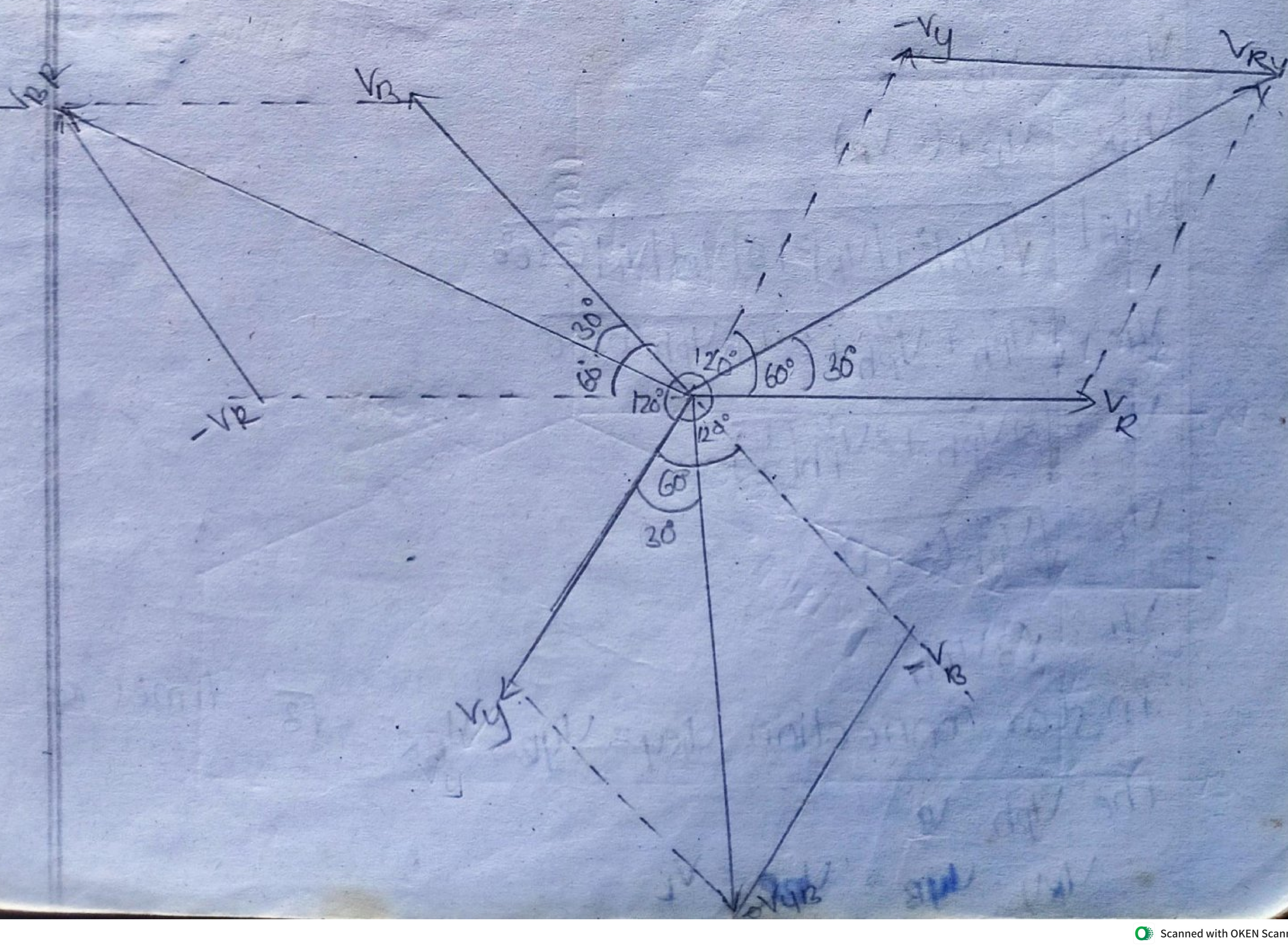
The relation between line & phase voltages



Let V_R, V_B, V_Y be the represents the phase voltages and the line voltages are V_{RY}, V_{YB}, V_{BR}

Let $|V_R| = |V_Y| = |V_B| = V_{ph}$

The line voltage V_{RY} is the vector difference of the V_R & V_Y or the vector sum of V_R & V_Y (reversed)



$V_{RY} = V_R - V_Y$

$V_{RY} = V_R + (-V_Y)$

$V_{RY} = \sqrt{|V_R|^2 + |V_Y|^2 + 2|V_R||V_Y|\cos\theta}$

$V_{RY} = \sqrt{V_{ph}^2 + V_{ph}^2 + 2|V_{ph}||V_{ph}|\cos 60^\circ}$

$V_L = \sqrt{2V_{ph}^2 + 2V_{ph}^2\left(\frac{1}{2}\right)}$

$V_L = \sqrt{V_{ph}^2(2+1)}$

$V_L = \sqrt{3}V_{ph}$

$V_{YB} = V_Y - V_B$

$V_{YB} = V_Y + (-V_B)$

$|V_{YB}| = \sqrt{|V_Y|^2 + |V_B|^2 + 2|V_Y||V_B|\cos\theta}$

$V_L = \sqrt{V_{ph}^2 + V_{ph}^2 + 2V_{ph}V_{ph}\cos 60^\circ}$

$V_L = \sqrt{2V_{ph}^2 + 2V_{ph}^2\left(\frac{1}{2}\right)}$

$V_L = \sqrt{V_{ph}^2(2+1)}$

$V_L = \sqrt{3}\, V_{ph}$

$V_{BR} = V_B - V_R$

$V_{BR} = V_B + (-V_R)$

$|V_{BR}| = \sqrt{|V_B|^2 + |V_R|^2 + 2|V_B||V_R|\cos 60^\circ}$

$V_L = \sqrt{V_{ph}^2 + V_{ph}^2 + 2V_{ph}V_{ph}\cos 60^\circ}$

$V_L = \sqrt{2V_{ph}^2 + 2V_{ph}^2\left(\frac{1}{2}\right)}$

$V_L = \sqrt{V_{ph}^2(2+1)}$

$V_L = \sqrt{3}V_{ph}$

In star connection $V_{RY} = V_{YB} = V_{BR} = \sqrt{3}$ times of the V_{ph} V_ϕ

$V_{RY} = V_{YB} = V_{BR} = V_L$

$\therefore V_L = \sqrt{3} V_{ph}$

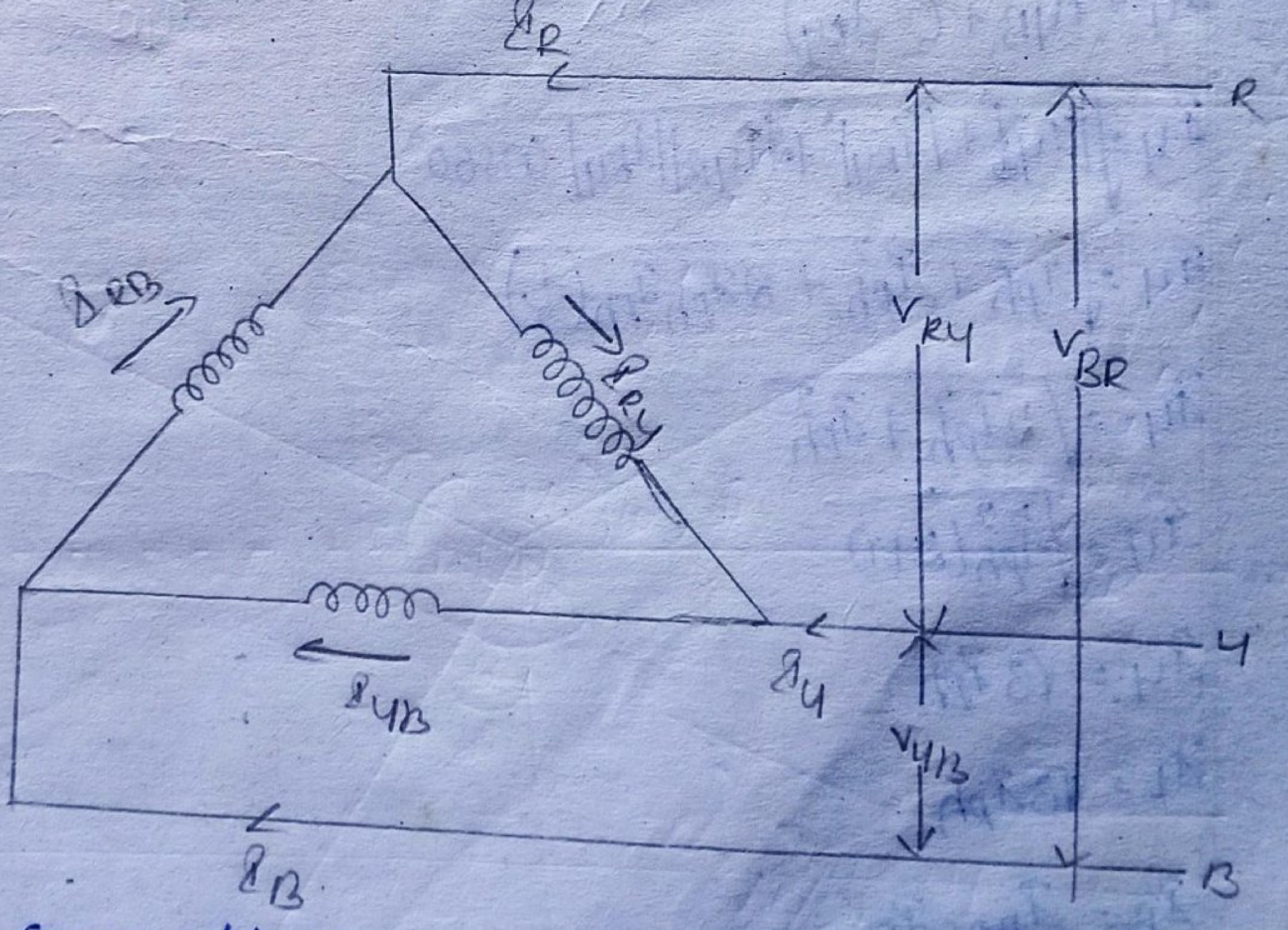
- In star connection each line in series with its individual phase winding
- Hence the line current in each line is same that current in the each phase winding
- let current in the line R be I_R; current in the line Y be I_Y, current in the line B be I_B
- $I_R = I_Y = I_B = I_{ph}$
- $\therefore I_L = I_{ph}$

$\therefore$ The line current = phase current

- The line voltages are 120° a part from each other & phase voltage also 120° a part from each other
- In balance star connection

$$I_R + I_Y + I_B = 0$$

Delta connection:-



In delta connection the currents in each line is the vector difference two phase currents flowing through the line

* The vector difference of corresponding phase currents
* Assume the three phase currents in delta are I_{RY}, I_{YB}, I_{BR} and the line current I_R, I_Y, I_B in
* In delta connection

$$|I_{RY}| = |I_{YB}| = |I_{BR}| = I_{ph}$$

* The line current I_R is vector difference of the I_{RY} & I_{BR}

$$I_R = I_{RY} - I_{BR}$$

$$I_R = I_{RY} + (-I_{BR})$$

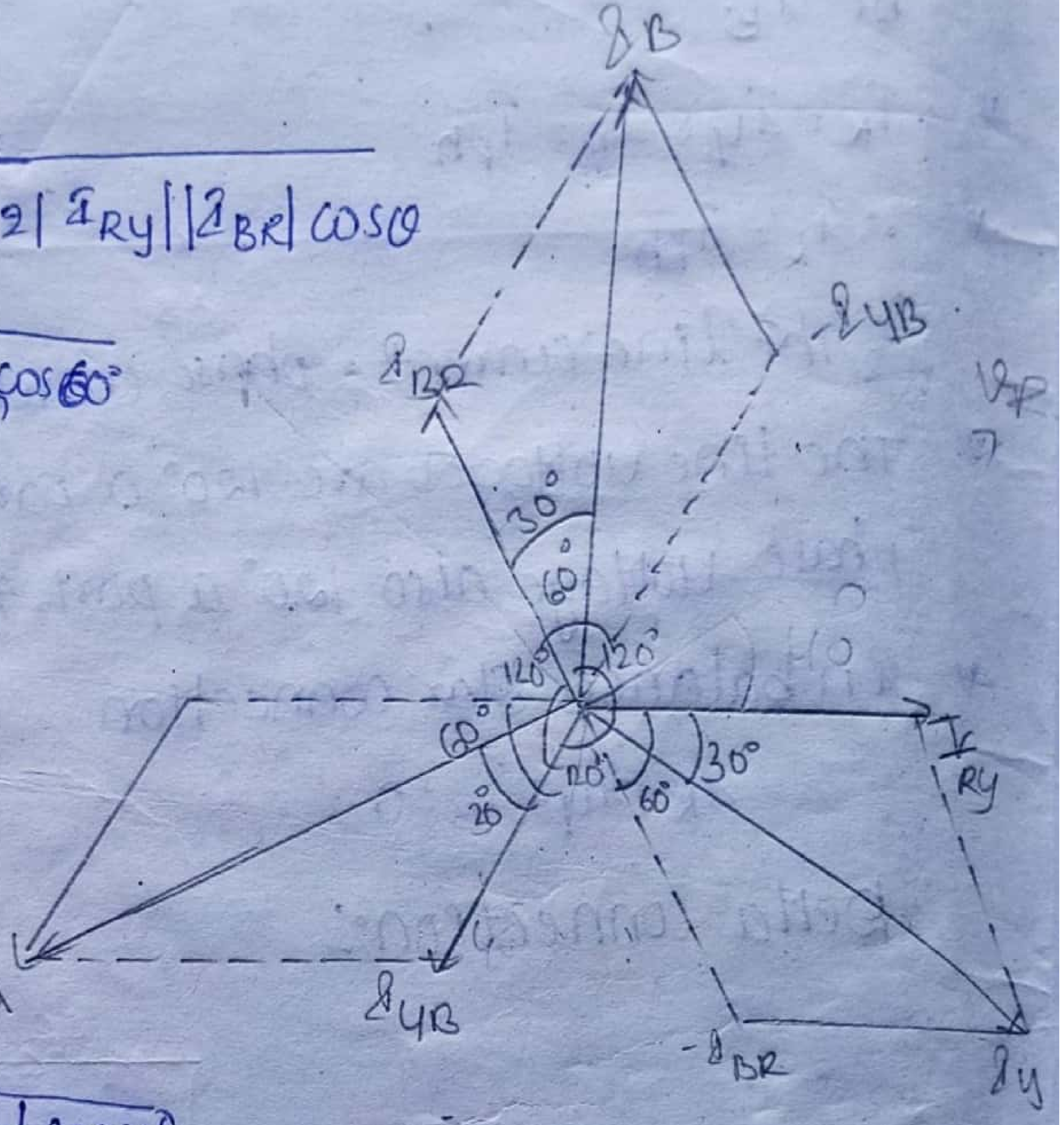
$$I_R = \sqrt{|I_{RY}|^2 + |I_{BR}|^2 + 2|I_{RY}||I_{BR}|\cos\theta}$$

$$I_R = \sqrt{I_{ph}^2 + I_{ph}^2 + 2I_{ph}I_{ph}\cos 60^\circ}$$

$$I_R = \sqrt{2I_{ph}^2 + 2I_{ph}^2 \left(\tfrac{1}{2}\right)}$$

$$I_R = \sqrt{I_{ph}^2 (2+1)}$$

$$I_R = \sqrt{3} I_{ph}$$



$$I_Y = I_{YB} - I_{RY}$$

$$I_Y = I_{YB} + (-I_{RY})$$

$$I_Y = \sqrt{|I_{YB}|^2 + |I_{RY}|^2 + 2|I_{YB}||I_{RY}|\cos 60^\circ}$$

$$I_Y = \sqrt{I_{ph}^2 + I_{ph}^2 + 2I_{ph}I_{ph}\left(\tfrac{1}{2}\right)}$$

$$I_Y = \sqrt{2I_{ph}^2 + I_{ph}^2}$$

$$I_Y = \sqrt{I_{ph}^2 (2+1)}$$

$$I_Y = \sqrt{3} I_{ph}$$

$$I_L = \sqrt{3} I_{ph}$$

$$I_B = I_{BR} - I_{YB}$$

$$I_B = \sqrt{|I_{BR}|^2 + |I_{YB}|^2 + 2|I_{BR}||I_{YB}|\cos\theta}$$

$$I_L = \sqrt{I_{ph}^2 + I_{ph}^2 + 2I_{ph}I_{ph}\cos 60}$$

$$I_L = \sqrt{2I_{ph}^2 + 2I_{ph}^2\left(\frac{1}{2}\right)}$$

$$I_L = \sqrt{I_{ph}^2(2+1)}$$

$$I_L = \sqrt{3}\,I_{ph}$$

In delta connection $I_R = I_Y = I_B = I_L$

$\therefore I_L = I_R = I_Y = I_B = \sqrt{3}\,I_{ph}$

$I_L = \sqrt{3}\,I_{ph}$

- In delta connection there is no neutral point. so, phase voltage is same as that of line voltage.

$V_{RY} = V_{YB} = V_{BR} = V_{ph}$

$V_L = V_{ph}$

- In delta connection the line currents are $120°$ part from each other & phase currents also $120°$ a part from each other.

Power in star connection

The total a active power or True power in 3-phase load is the sum of power in the 3 phases for a balanced load. the power in each load is the same. Hence total power = 3 power in each phase.

$P = VI\cos\phi$ /phase $\quad \phi$ is the angle b/w voltage & current

$P_{1\phi} = 1 \times V_{ph} I_{ph} \cos\phi$

$P_{3\phi} = 3 V_{ph} I_{ph} \cos\phi$

In star connection

$V_L = \sqrt{3}\,V_{ph} \quad I_L = I_{ph}$

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

$P_{3\phi} = 3\frac{V_L}{\sqrt{3}} I_L \cos\phi$

$P_{3\phi} = \sqrt{3} V_L I_L \cos\phi$

it is the active power

$P_{3\phi} = \sqrt{3} V_L I_L \cos\phi$ k.w

Reactive power

$Q = VI \sin\phi$

$Q_{3\phi} = 3 V_{ph} I_{ph} \sin\phi$

$Q_{3\phi} = \frac{V_L}{\sqrt{3}} 3 I_L \sin\phi$

$Q_{3\phi} = \sqrt{3} V_L I_L \sin\phi$ KVAR.

Apparent power

$S = VI$

$S_{3\phi} = 3 V_{ph} I_{ph}$

$= 3 \frac{V_L}{\sqrt{3}} I_L$

$= \sqrt{3} V_L I_L = \sqrt{3} V_L I_L$ KVA

power in delta connection

$P = VI \cos\phi$

In delta connection

$I_L = \sqrt{3} I_{ph}$ $\quad I_{ph} = \frac{I_L}{\sqrt{3}}$

$V_L = V_{ph}$

$P_{3\phi} = 3 V_{ph} I_{ph} \cos\phi$

$= 3 V_L \frac{I_L}{\sqrt{3}} \cos\phi$

$= \sqrt{3} V_L I_L \cos\phi$

True power

$P_{3\phi} = \sqrt{3} V_L I_L \cos\phi$ k.w

$\sqrt{3} V_L$

$P_L = V_L I_L \cos\phi$

$\frac{I_L}{\sqrt{3}} = \sqrt{3} I_{ph}$

$I_{ph} = \frac{I_L}{\sqrt{3}}$

$V_L = V_{ph}$

$P_{3\phi} = 3 V_{ph} I_{ph} \cos$

Reactive power

$Q = VI\sin\phi$

$Q_{3\phi} = 3V_{ph}I_{ph}\sin\phi$

$Q_{3\phi} = 3V_L\frac{I_L}{\sqrt{3}}\sin\phi$

$Q_{3\phi} = \sqrt{3}V_L I_L\sin\phi$ (KVAR)

In delta connection the total power in the delta circuit is the sum of the power in the 3ϕ's the load is balance. The power consumed in each phase is same the total power = 3 × power in each phase

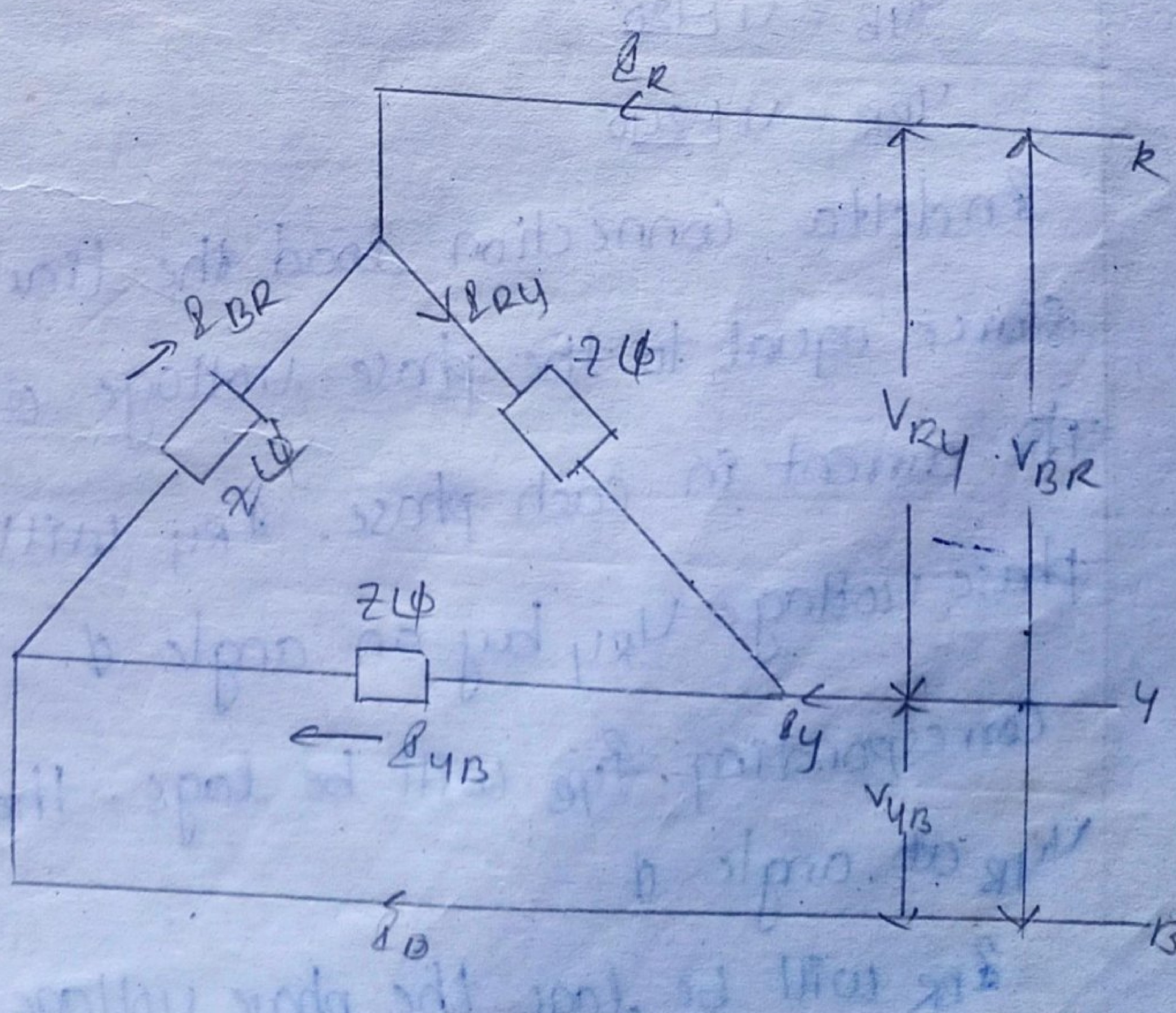
Apparent power

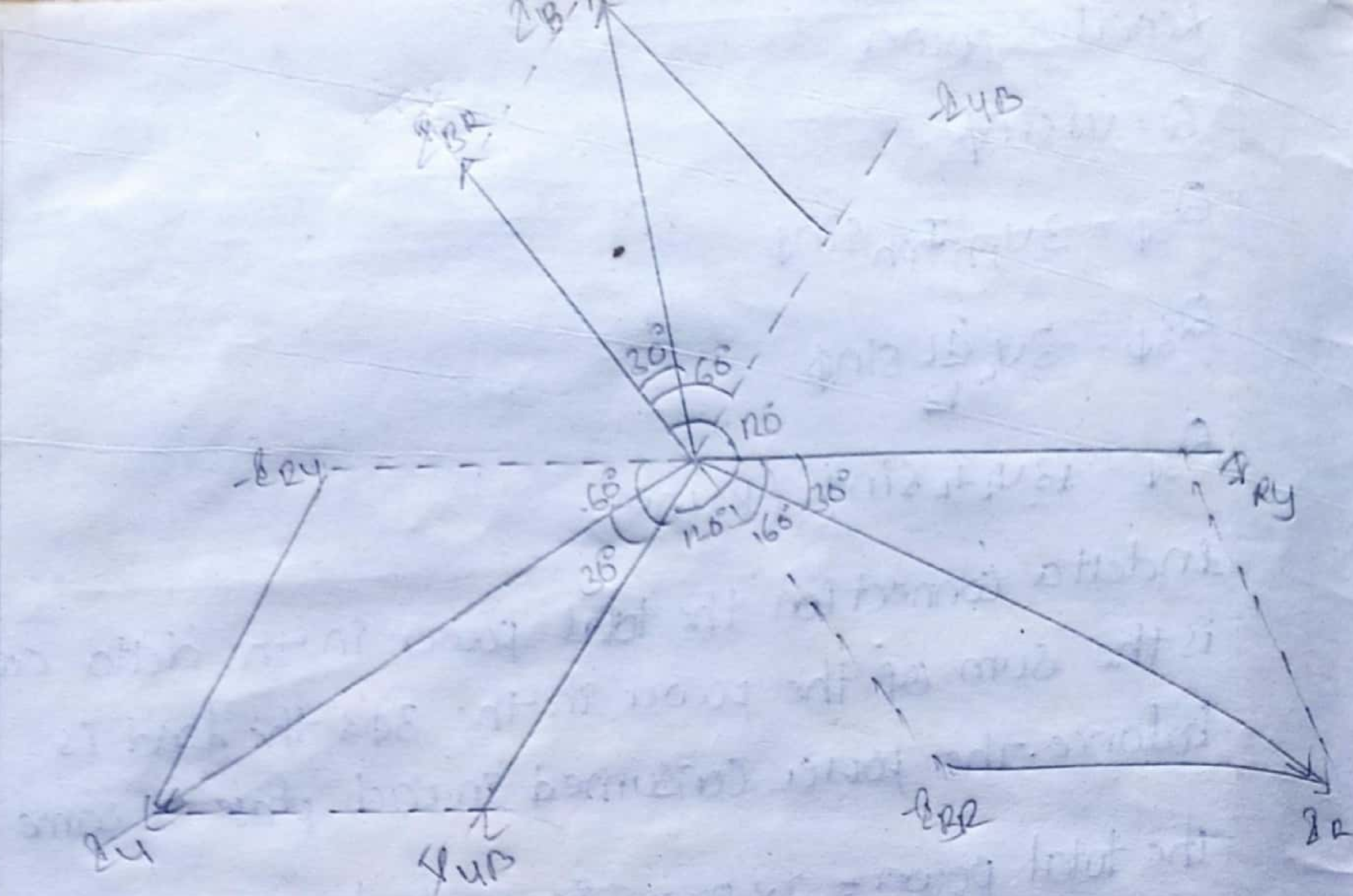
$S_{3\phi} = 3V_{ph}I_{ph}$

$S_{3\phi} = 3V_L\frac{I_L}{\sqrt{3}}$

$= \sqrt{3}V_L I_L$ KVA

3-phase balanced delta connection





A 3ϕ 3wire balance system supplying power to a balanced 3ϕ delta load the phase sequence is R, Y, B. Let us assume the line voltages $V_{RY} = V\angle 0^\circ$ as the reference phasor then the three source voltages are given by

$V_{RY} = V\angle 0^\circ$

$V_{YB} = V\angle -120^\circ$

$V_{BR} = V\angle -240^\circ$

In delta connection load the line voltage of the source equal to the phase voltage of the load. The current in each phase. I_{RY} will be lags the phase voltage V_{RY} by an angle ϕ

Corresponding I_{YB} will be lags the phase voltage V_{YB} at angle ϕ

I_{BR} will be lags the phase voltage V_{BR} at an angle ϕ. If the load impedence is Z at angle ϕ

The current flowing in the 3 load impedence are

$$I_{RY} = \frac{V_{RY}\angle 0^\circ}{Z\angle\phi} = \frac{V\angle 0^\circ}{Z\angle\phi} = \left|\frac{V}{Z}\right|\angle -\phi$$

R (-)

$$I_{YB} = \frac{V_{YB}\angle -120^\circ}{Z\angle\phi} = \frac{V\angle -120^\circ}{Z\angle\phi} = \left|\frac{V}{Z}\right|\angle -120^\circ - \phi$$

$$I_{BR} = \frac{V_{BR}\angle -240^\circ}{Z\angle\phi} = \frac{V\angle -240^\circ}{Z\angle\phi} = \left|\frac{V}{Z}\right|\angle -240^\circ - \phi$$

The line currents are $\sqrt{3}$ ph from phasor diagram 30° behind. they respective phase currents in the line R is given by

$$I_R = \sqrt{3}\, I_{RY}$$

$$= \sqrt{3}\left|\frac{V}{Z}\right|\angle -\phi - 30^\circ$$

$$I_Y = \sqrt{3}\, I_{YB}$$

$$= \sqrt{3}\left|\frac{V}{Z}\right|\angle -120^\circ - \phi$$

$$I_B = \sqrt{3}\, I_{BR}$$

$$= \sqrt{3}\left|\frac{V}{Z}\right|\angle -240^\circ - \phi^\circ$$

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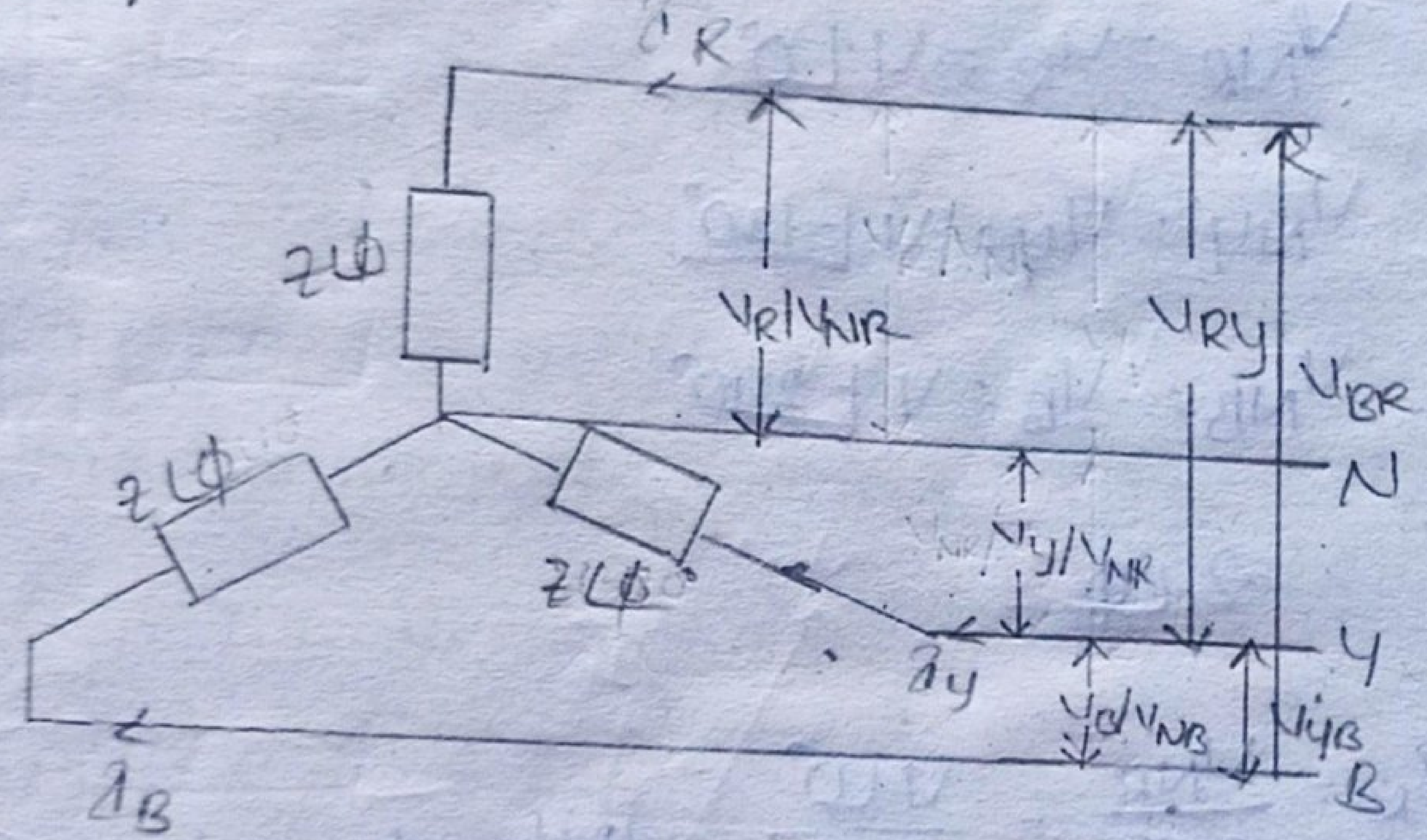
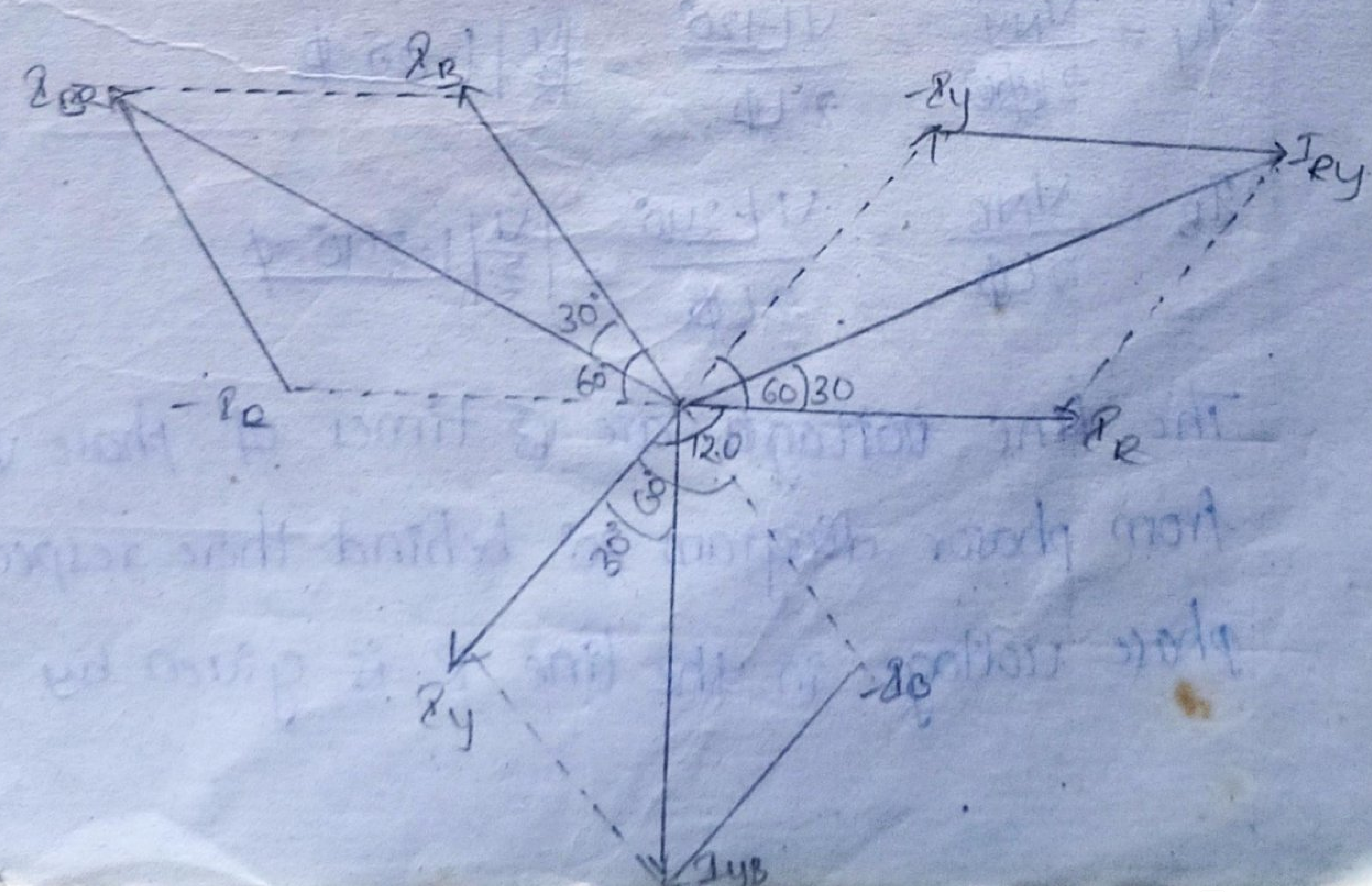
Balanced 3-phase star connected load:-





Consider a 3-phase 3 wire system supplying a power to a balance 3-phase star connected load the phase sequence is R, Y, B

In star connection phase current equal to line current

i.e $I_{ph} = I_L$

The three-line or phase currents I_R, I_Y, I_B

The phase voltages V_R, V_Y, V_B (or) V_{NR}, V_{NY}, V_{NB} in between any line & neutral

Let us assume $V_{RN} = V\angle 0^\circ$ as reference phasor

$Z\angle\phi$ is the load impedence.

$V_{NR} = V_R = V\angle 0^\circ$

$V_{NY} = V_Y = V\angle -120^\circ$

$V_{NB} = V_B = V\angle -240^\circ$

$Z\angle\phi$

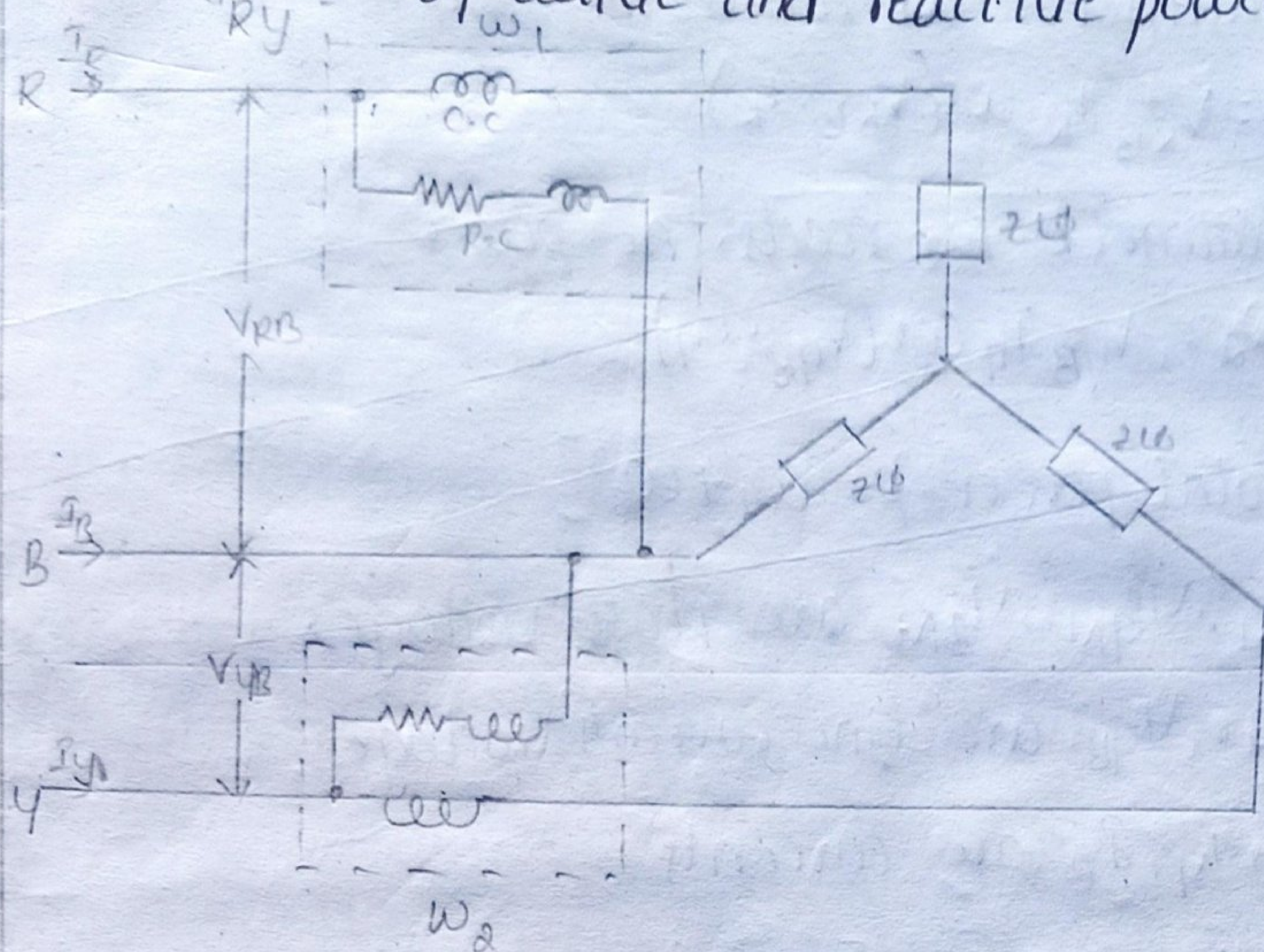
$$I_R = \frac{V_{NR}}{Z\angle\phi} = \frac{V\angle 0^\circ}{Z\angle\phi} = \left|\frac{V}{Z}\right| \angle -\phi$$

$$I_Y = \frac{V_{NY}}{Z\angle\phi} = \frac{V\angle -120^\circ}{Z\angle\phi} = \left|\frac{V}{Z}\right| \angle -120^\circ-\phi$$

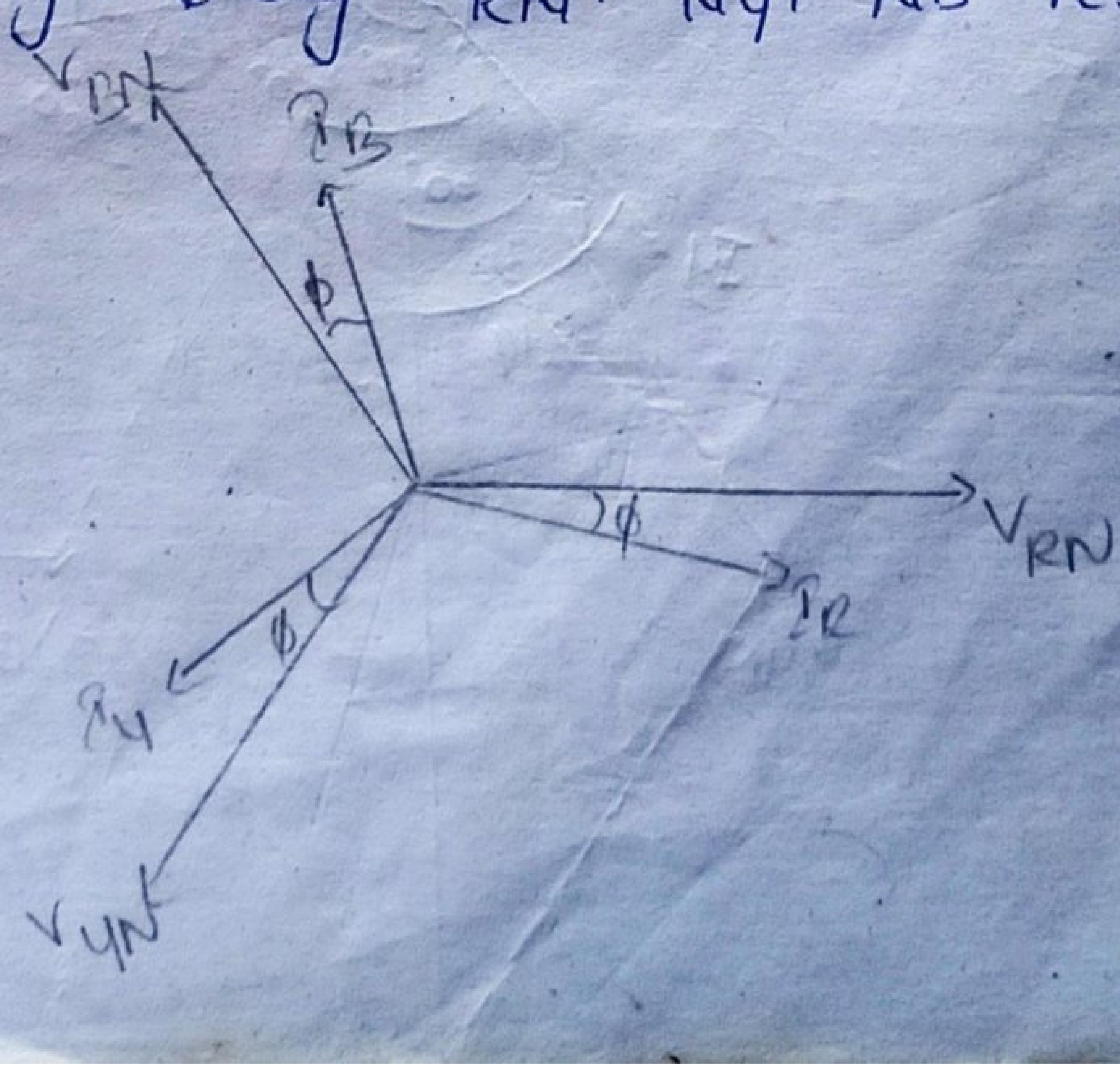
$$I_B = \frac{V_{NB}}{Z\angle\phi} = \frac{V\angle -240^\circ}{Z\angle\phi} = \left|\frac{V}{Z}\right| \angle -240^\circ-\phi$$

The line voltages are $\sqrt{3}$ times of phase voltages from phasor diagram 30° behind there respective phase voltage in the line R is given by

Measurement of active and reactive power:-



* Two wattmeter are connected in are shown in figure for star connected load.
* Two current coils are connected to a any pair of lines and potential coil is connected to a third line
* The algebric sum of the 2 wattmeter will gives the total power three phase load whether load is balanced
* The line voltages are V_{RB} & V_{YB}.
* The line currents are I_R & I_Y & I_B
* The phase voltages are V_R, V_Y, V_B or V_{NR}, V_{NY}, V_{NB}
* Assume the load is logging load. the currents I_R, I_Y, I_B loggs ~~curr an angle ϕ by~~ V_{RN}, V_{NY}, V_{NB} respectively



* The wattmeter W_1 reads the power

$$W_1 = V_{RB} I_R \cos(V_{RB} \cdot I_R)$$

* The wattmete W_2 reads the power

$$W_2 = V_{YB} I_Y \cos(V_{YB} \cdot I_Y)$$

* The total power $P = W_1 + W_2$

V_{RN}, V_{YN}, V_{BN} are phase voltages

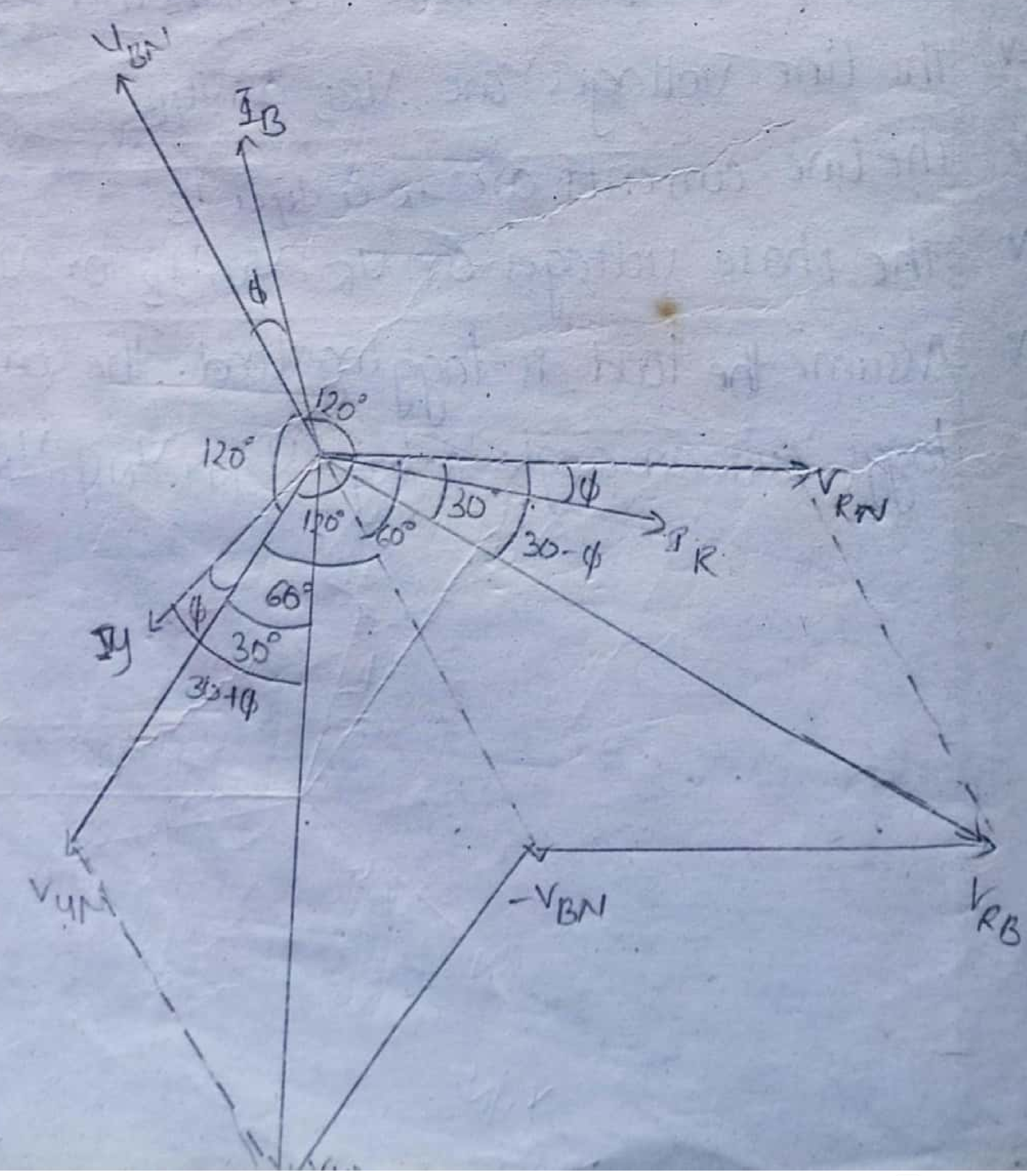
V_{RB}, V_{YB} are line ~~currents~~ voltages

I_R, I_Y, I_B are currents

i.e $I_R = I_Y = I_B = I_L = I_{ph}$

$$V_{RB} = V_{RN} - V_{BN}$$
$$= V_{RN} + (-V_{BN})$$

$$V_{YB} = V_{YN} - V_{BN}$$
$$= V_{YN} + (-V_{BN})$$



The angle between V_{RN} & I_R is ϕ,
V_{YN} & I_Y is ϕ

$$V_{RN} = V_{YN} = V_{BN} = V_{ph}$$

$$V_{RB} = V_{YB} = V_L ,$$

$$I_R = I_Y = I_B = I_L = I_{ph}$$

From phasor diagram the angle between V_{RB} & I_R is $30-\phi$
The angle between V_{YB} & I_Y is $30+\phi$

The wattmeter W_1 reads the power

$$W_1 = V_{RB} I_R \cos(V_{RB} \hat{} I_R)$$
$$= V_{RB} I_R \cos(30-\phi)$$

The wattmeter W_2 reads the power

$$W_2 = V_{YB} I_Y \cos(V_{YB} \hat{} I_Y)$$
$$= V_{YB} I_Y \cos(30+\phi)$$

$$W_1 = P_1 = V_{RB} I_R \cos(V_{RB} \hat{} I_R)$$
$$W_1 = P_1 = V_L I_L \cos(30-\phi)$$
$$W_2 = P_2 = V_{YB} \cdot I_Y \cos(V_{YB} \hat{} I_Y)$$
$$W_2 = P_2 = V_L I_L \cos(30+\phi)$$

Total power $W = W_1 + W_2$

$$= V_L I_L \cos(30-\phi) + V_L I_L \cos(30+\phi)$$

A $$= V_L I_L [\cos(30-\phi) + \cos(30+\phi)]$$

Active power $p = V_L I_L [\cos 30 \cdot \cos\phi + \cancel{\sin 30 \sin\phi} + \cos 30 \cdot \cos\phi - \cancel{\sin 30 \sin\phi}]$

$$P = V_L I_L \, 2\cos 30 \cdot \cos\phi$$
$$P = V_L I_L \, \cancel{2} \cos\phi \frac{\sqrt{3}}{\cancel{2}}$$
$$P = \sqrt{3} V_L I_L \cos\phi$$

$\because \cos(C-D) = \cos C \cos D + \sin C \sin D$

$\cos(C+D) = \cos C \cos D - \sin C \sin D$

Reactive power $Q = W_1 - W_2$

$Q = V_L I_L \cos(30-\phi) - V_L I_L \cos(30+\phi)$

$Q = V_L I_L [\cos(30-\phi) - \cos(30+\phi)]$

$Q = V_L I_L [\cos 30 . \cancel{\cos\phi} + \sin 30 \sin\phi - \cancel{\cos 30 \cos\phi} + \sin 30 \sin\phi]$

$Q = V_L I_L \, 2 \sin 30 . \sin\phi$

$Q = V_L I_L \cancel{2} \sin\phi \frac{1}{\cancel{2}}$

$Q = V_L I_L \sin\phi$

But we know

$Q = \sqrt{3} V_L I_L \sin\phi$

$Q = \sqrt{3}[W_1 - W_2]$

Power factor:-

To find out the power factor by using two wattmeter

W.K.T

$P = W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi \longrightarrow (1)$

$Q = W_1 - W_2 = V_L I_L \sin\phi \longrightarrow (2)$

$\frac{(2)}{(1)} \Rightarrow \frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin\phi}{\sqrt{3} V_L I_L \cos\phi}$

$\frac{W_1 - W_2}{W_1 + W_2} = \frac{1}{\sqrt{3}} \tan\phi$

$\tan\phi = \sqrt{3}\left[\frac{W_1 - W_2}{W_1 + W_2}\right]$

$\phi = \tan^{-1}\left[\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)\right]$

pb:1 Two wattmeter are used to measure power in a 3-ϕ 3 wire load. Determine the total power, power facto and reactive power. If two wattmeters are read i, 1000 watts each both positive ii, 1000 watts each but opposite sign.

w_1 & $w_2 = 1000$

$w_1 + w_2 = 1000 + 1000$
$= 2000$

$w_1 - w_2 = 1000 - 1000 = 0$

Given

i, $w_1 = 1000w$, $w_2 = 1000w$

both are positive

ii, Total power $p = w_1 + w_2 = 2000w$

iii, Reactive power $Q = \sqrt{3}(w_1 - w_2) = \sqrt{3}(1000 - 1000) = 0$

iv, power factor $\cos\phi = ?$

phase angle $\phi = \tan^{-1}\left[\frac{\sqrt{3}(w_1 - w_2)}{w_1 + w_2}\right] = \tan^{-1}\left(\frac{\sqrt{3}(0)}{2000}\right) = 0$

$\phi = 0$

ii, Opposite sign.

i, $w_1 = 1000w$; $w_2 = -1000w$

Total power $p = w_1 - w_2$
$= 1000 - 1000$
$= 0$.

Reactive power $Q = \sqrt{3}(w_1 - w_2)$
$= \sqrt{3}(1000 - (-1000))$
$= \sqrt{3}(2000) = 3464.1$

$w_1 = +1000$
$w_2 = -1000$
$Q = \sqrt{3}(w_1 - w_2)$
$= \sqrt{3}(w_1 - (-w_2))$
$= \sqrt{3}(1000 - (-1000))$

power factor $\cos\phi = ?$

phase angle $\phi = \tan^{-1}\left(\frac{\sqrt{3}(w_1 - w_2)}{w_1 + w_2}\right)$

$\phi = \tan^{-1}\left(\frac{\sqrt{3}(1000 - (-1000))}{1000 - 1000}\right)$

$$\phi = Tan^{-1}\left(\frac{\sqrt{3}(2000)}{0}\right)$$

$$\phi = Tan^{-1}\infty$$

$$\phi = Tan^{-1}Tan 90$$

$$\phi = 90^\circ$$

$\cos\phi = \cos 90^\circ = 0$

pb: 2 The power delivered to a balanced delta connected load a ~~t~~ 400 volts 3-ϕ supply is measured by a two wattmeter methods. if the reads of the two wattmeters are 2000 watts 1500 watts respectively. calculate magnitude of the impedence. in each phase arm of the delta connected load & its resistive componed.

Given.

Delta connected load

$V_L = 400V$ - 3-ϕ

$f = 50Hz$

$P_1 = W_1 = 2000W$

$P_2 = W_2 = 1500W$

$Z = ?$

$R = ?$

The phase angle $\phi = \tan^{-1}\left(\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right)$

$$= \tan^{-1}\left(\frac{\sqrt{3}(2000-1500)}{2000+1500}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{3}(5\cancel{00})}{\cancel{35}\cancel{00}_{7}}\right)$$

$$= 13.49^\circ$$

$\cos\phi = 0.97^\circ$

$P = \sqrt{3} V_L I_L \cos\phi$

$P = W_1 + W_2 = 3500\ W$

$I_L = \dfrac{P}{\sqrt{3} V_L \cos\phi} = \dfrac{3500}{\sqrt{3} \times 400 \times (0.97)}$

$P = \sqrt{3} V_L I_L \cos\phi$

$I_L = \dfrac{P}{\sqrt{3} V_L \cos\phi}$

$\boxed{\sqrt{3} V_L = I_L Z}$

$I_L = 5.2080$ Amp.

In delta connected

$\sqrt{3} V_L = I_L Z$

$Z = \dfrac{\sqrt{3} V_L}{I_L}$

$Z = 133.08\ \Omega$

The relation b/w Z_ϕ & R_ϕ is from $\cos\phi = \dfrac{R}{Z}$

$R = Z \cos\phi$

$R = 129.08\ \Omega$

20-9-22

Po:3 A balance 3-ϕ star connected load of 150kW. takes a leading current of 100 amp. with a line voltage of 1100 volts, 50Hz find the circuit constants of the load for phase

Given

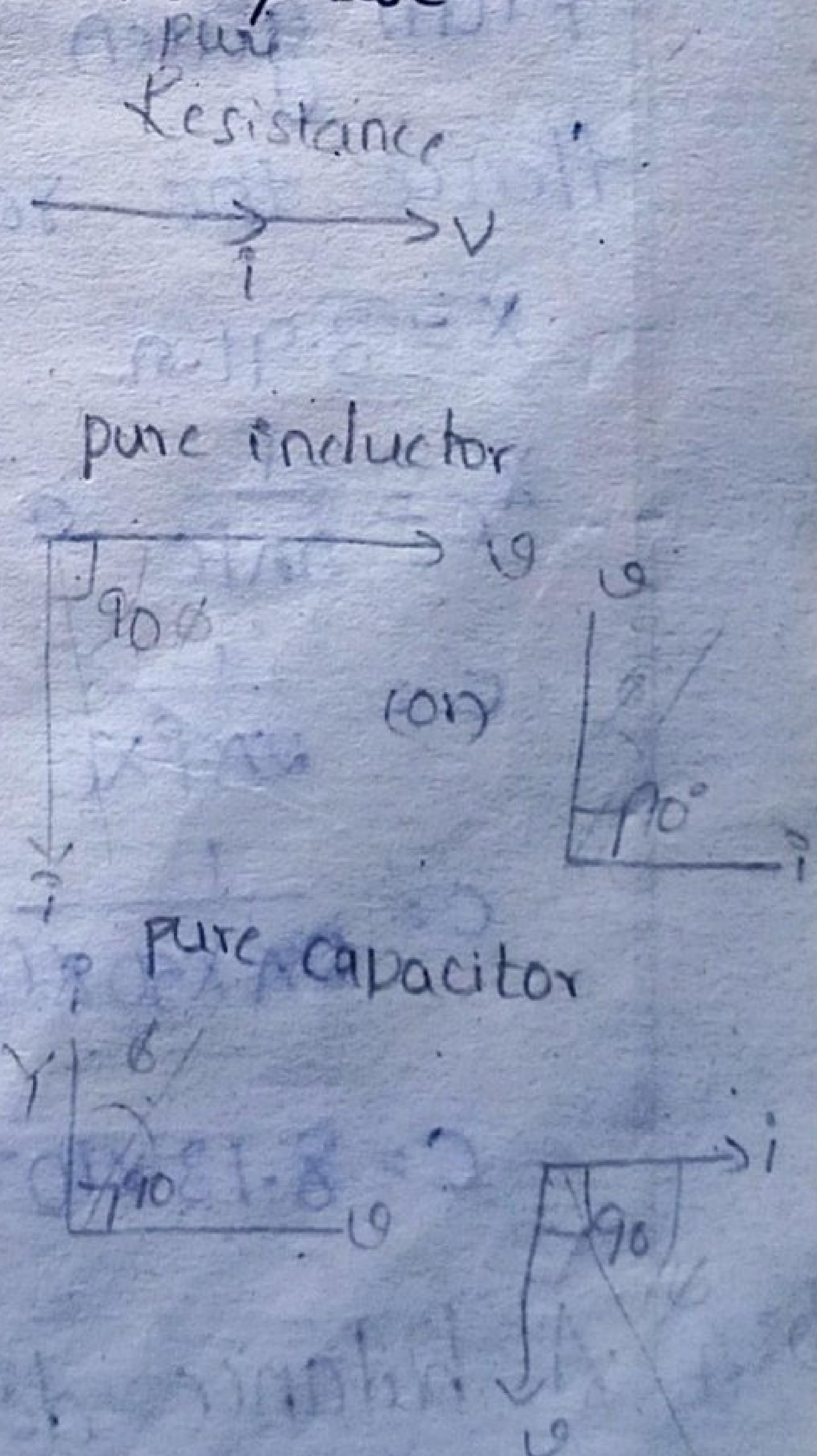
$f = 50Hz$

$V_L = 1100V$

$P = 150\ kW$

0.7879

load current $I_L = 100\ A$



$P = \sqrt{3} V_L I_L \cos\phi$

$\cos\phi = \dfrac{P}{\sqrt{3} V_L I_L}$

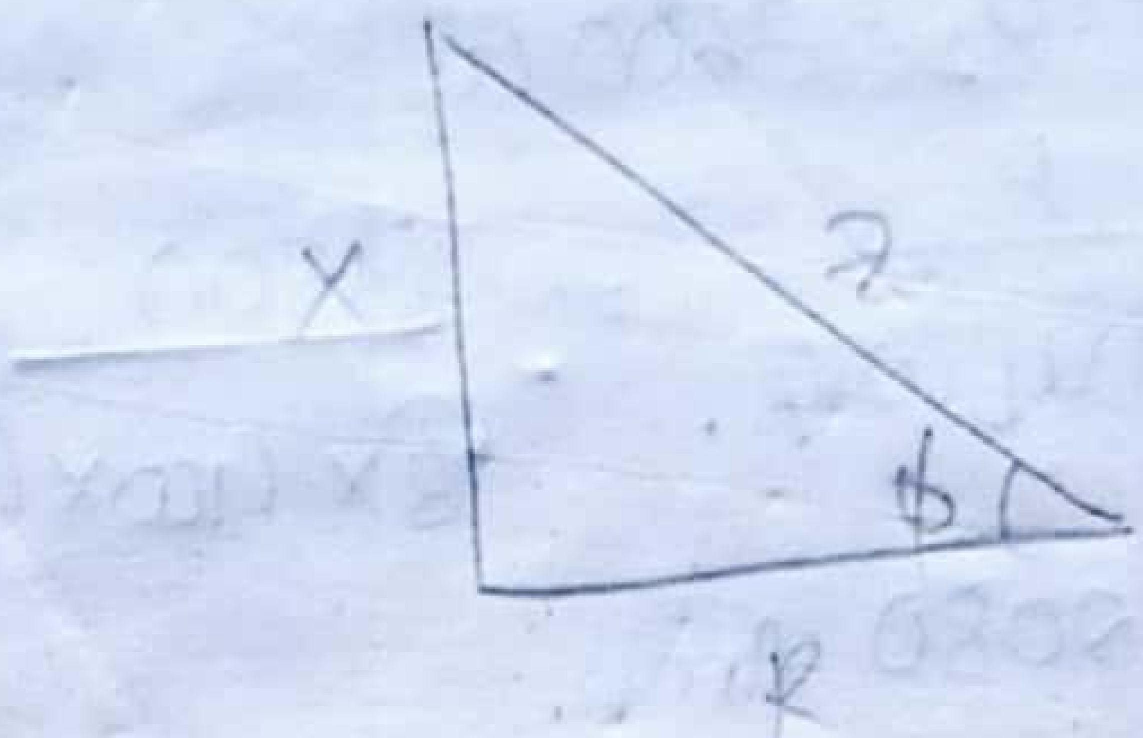
$\phi = \cos^{-1}\left(\dfrac{150 \times 10^{+3}}{\sqrt{3}(1100)(100)}\right)$

$\phi = 38.06$

$\cos\phi = 0.7872$.

$\cos\phi = \frac{R}{Z}$

$R = Z\cos\phi$



$Z = \frac{\sqrt{3}\, V_L}{\sqrt{3}\, I_L}$

$Z = \frac{1100}{\sqrt{3}\,100}$

$Z = 6.35\ \Omega$

In delta connection

$V_L = V_{ph}$

$I_L = \sqrt{3}\, I_{ph}$

In star

$I_L = I_{ph}$

$V_L = \sqrt{3}\, V_{ph}$

$R = Z\cos\phi$

$R = 6.35(0.7872)$

$R = 4.99\ \Omega$

$\sin\phi = \frac{X}{Z}$

$X = Z\sin\phi \qquad = (19.05)(0.35)$

$X = (6.35)(0.616)$

$X = 3.91\ \Omega$

From given problem current is leading component Hence the reactance is taken capacitive reactance

$X = 3.91\ \Omega \qquad X$ is taken as X_C

$X_C = \frac{1}{2\pi f C}\ \Omega \qquad X = X_C$

$C = \frac{1}{2\pi f X_C}$

$C = \frac{1}{2\pi \times 50 \times (3.91)}$

$C = 8.13 \times 10^{-4}$ Faraday

4 A balance delta connected load is supplied from a symmetrical 3ϕ 400V, 50Hz supply system. The current in each phase is 20A & lags behind its

phase voltage by an angle 40°

i, Line current   ii, Total power   iii, also draw the phasor diagram showing the voltages & currents in the lines & phases   iv, The wattmeter reading In two wattmeter are used.

sol Given data

delta connected load

$V_L = V_{ph} = 400V$ $\phi = 40°$

$f = 50Hz$

$I_{ph} = 20A$

i, line current $I_L = \sqrt{3} I_{ph}$

$I_L = \sqrt{3}(20)$

$I_L = 34.64$ amp

iii, Total power

$P = \sqrt{3} V_L I_L \cos\phi$

$Q = \sqrt{3} V_L I_L \sin\phi$

$\cos\phi$

$P = \sqrt{3} \times 400 \times 34.64 \times \cos 40°$

$P = 23999.2 \times \cos 40$

$P = 23999.2 \times (0.766)$

$P = 18384.4$ watts

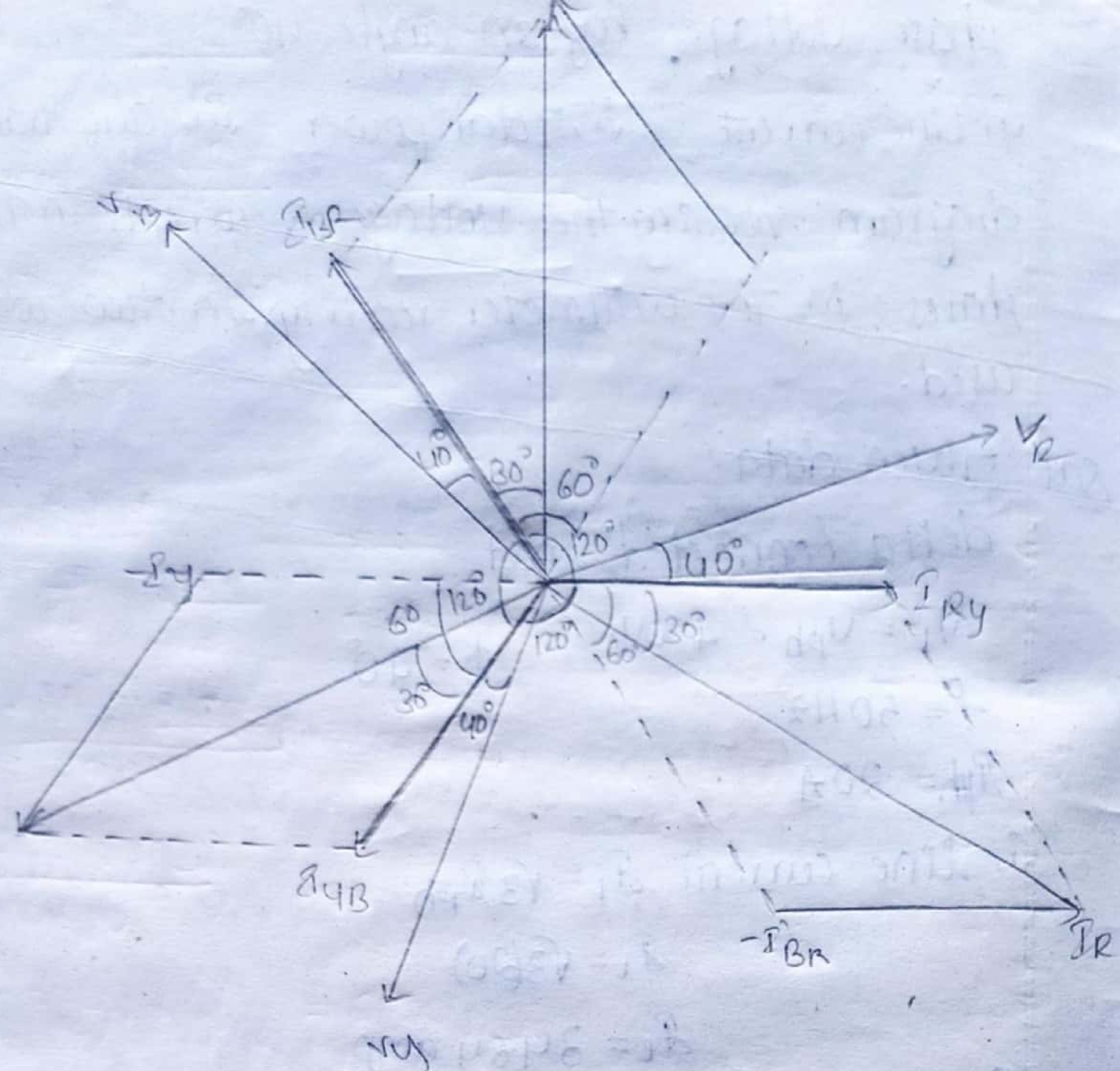
$Q = \sqrt{3} V_L I_L \sin\phi$

$= 23999.2 \times (0.\cancel{64}2)$

$= 15426.3$ watts

$S = \sqrt{3} V_L I_L$

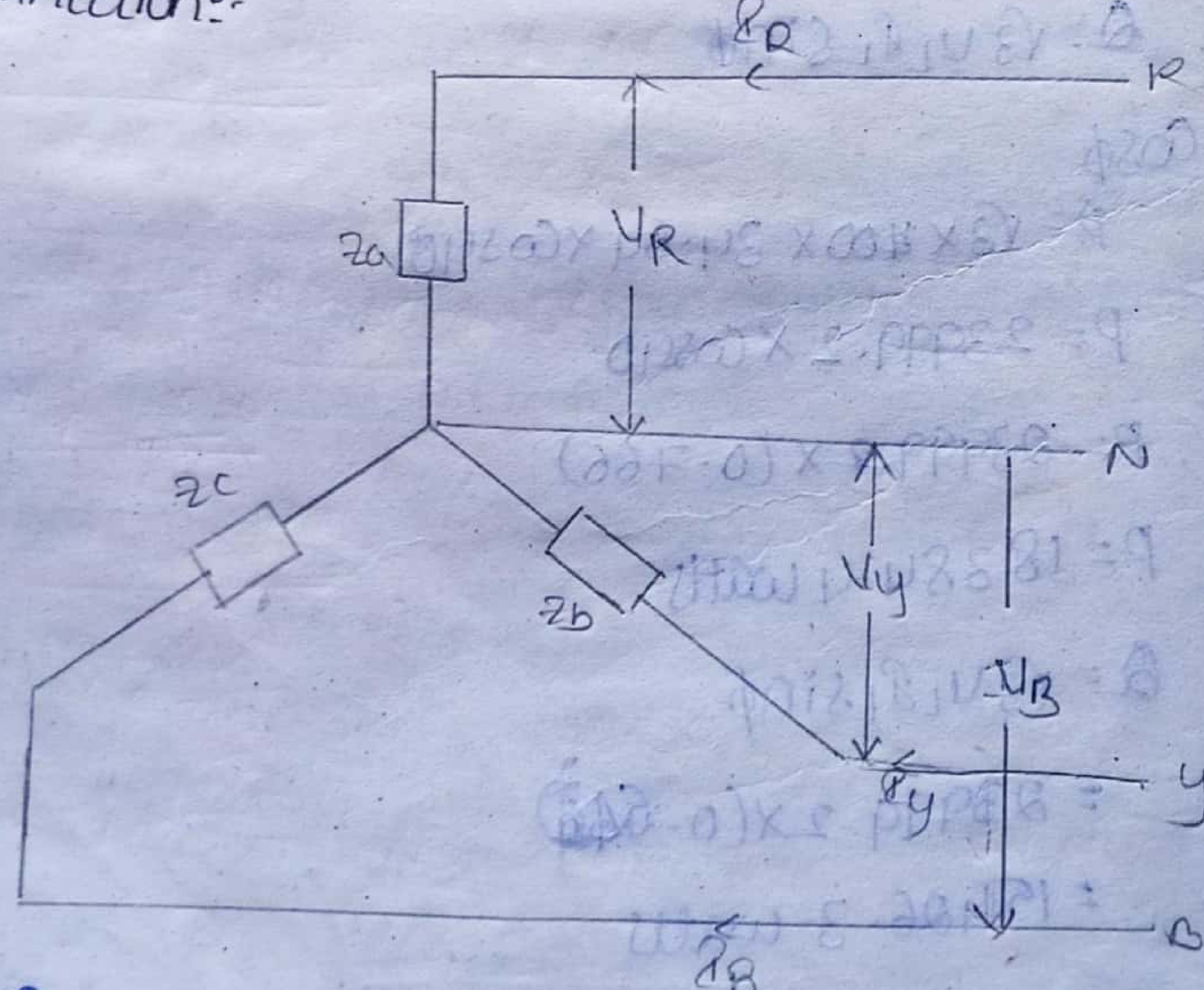
$S = 23999.2$ watt   57809.9



24-07-22

unbalanced 3-phase circuits

Star connection:-



$V_R = V_{ph}\angle 0°$

$V_Y = V_{ph}\angle -120°$

$V_B = V_{ph}\angle -240°$

4 - (1) 3

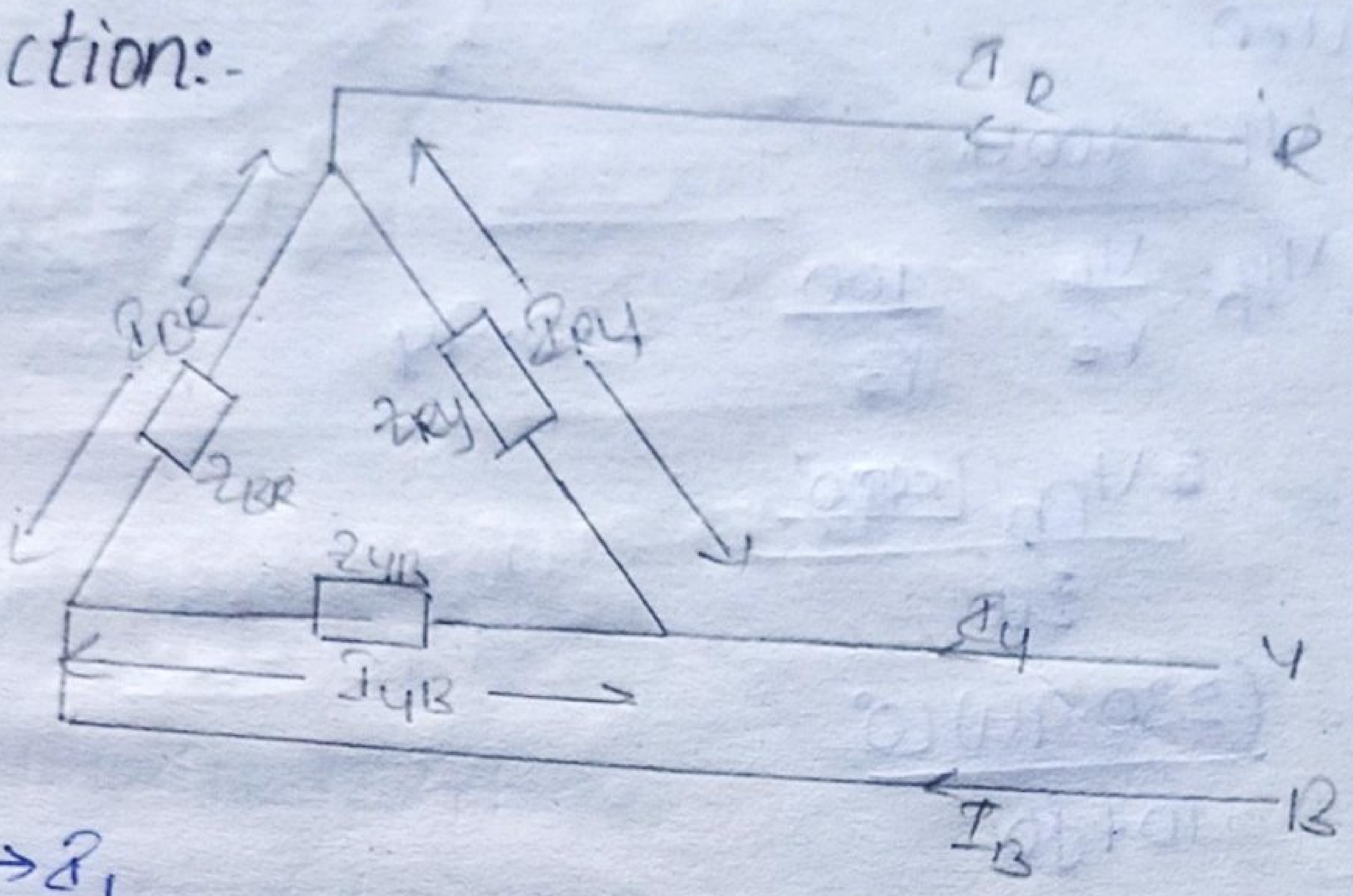
$I_R = \frac{V_R}{Z_a} = \frac{V_{ph}\angle 0^\circ}{Z_a}$

$I_N = -[I_R + I_Y + I_B]$

$I_Y = \frac{V_Y}{Z_b} = \frac{V_{ph}\angle -120^\circ}{Z_b}$

$I_B = \frac{V_B}{Z_c} = \frac{V_{ph}\angle -240^\circ}{Z_c}$

Delta connection:-



$I_R, I_Y, I_B \rightarrow I_L$

$I_{RY}, I_{YB}, I_{BR} \rightarrow I_{ph}$

$V_{RY} = V_{ph}\angle 0^\circ$

$V_{YB} = V_{ph}\angle -120^\circ$

$V_{BR} = V_{ph}\angle -240^\circ$

$I_{RY} = \frac{V_{RY}}{Z_a} = \frac{V_{ph}\angle 0^\circ}{Z_a}$

$I_{YB} = \frac{V_{YB}}{Z_b} = \frac{V_{ph}\angle -120^\circ}{Z_b}$

26-09-22 $I_{BR} = \frac{V_{BR}}{Z_c} = \frac{V_{ph}\angle -240^\circ}{Z_c}$

pb: A 3-phase 400volts 4-wire system has a star connected ~~load~~ load with $Z_a = (100 + j0)\Omega$, $Z_b = (15 + j10)\Omega$ $Z_c = (0 + j5)\Omega$ Find the line currents and current through Neutral point. draw the phasor diagram

I_R, I_Y, I_B are phase currents

In star connection

phase current $I_R = \frac{V_R\angle 0^\circ}{Z_a}$

$$I_Y = \frac{V_Y \angle -120^\circ}{Z_b} = \frac{V_{ph} \angle -120^\circ}{Z_b}$$

$$I_B = \frac{V_B \angle -240^\circ}{Z_c} = \frac{V_{ph} \angle -240}{Z_c}$$

In star connection

$I_P = I_L$.

Given

$V_L = 400V$.

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94$$

$$I_R = \frac{V_{ph} \angle 0^\circ}{Z_a}$$

$$= \frac{(230.94) \angle 0^\circ}{10 + j0}$$

$$= 23.09 + 0j = 23.09 \angle 0^\circ$$

$$I_Y = \frac{V_{ph} \angle -120^\circ}{Z_b}$$

$$= \frac{(230.94) \angle -120^\circ}{(15 + j10)}$$

$$= \cancel{23.09 + 10j}\ 12.535 + 1.06j = 12.75 \angle -153.69^\circ$$

$$I_B = \frac{V_{ph} \angle -240^\circ}{Z_c}$$

$$= \frac{(230.94) \angle -240^\circ}{(4 + 5j)}$$

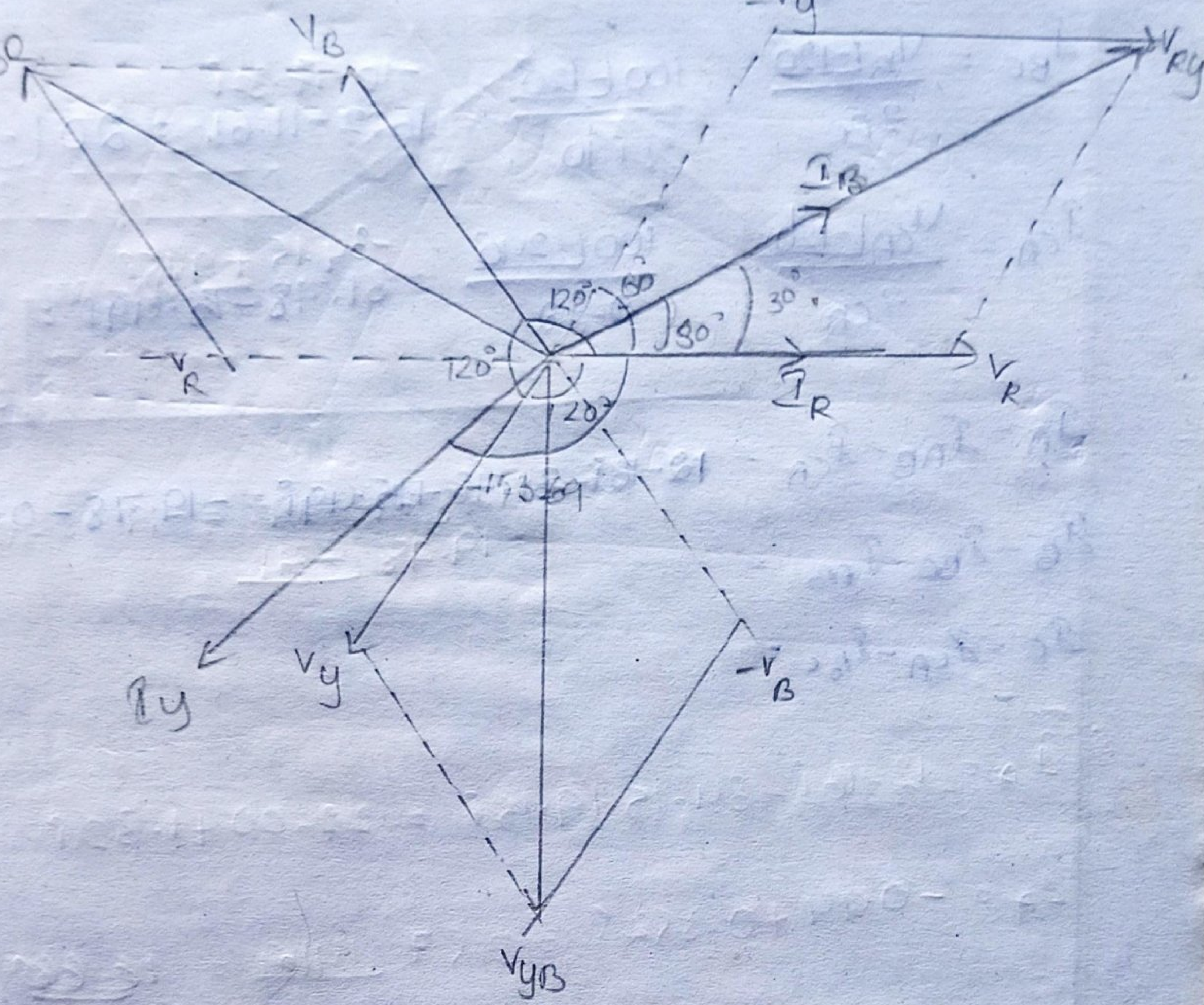
$$= 43.66 - 15.04j = 46 \angle 30^\circ$$

In star connection phase currents = line currents

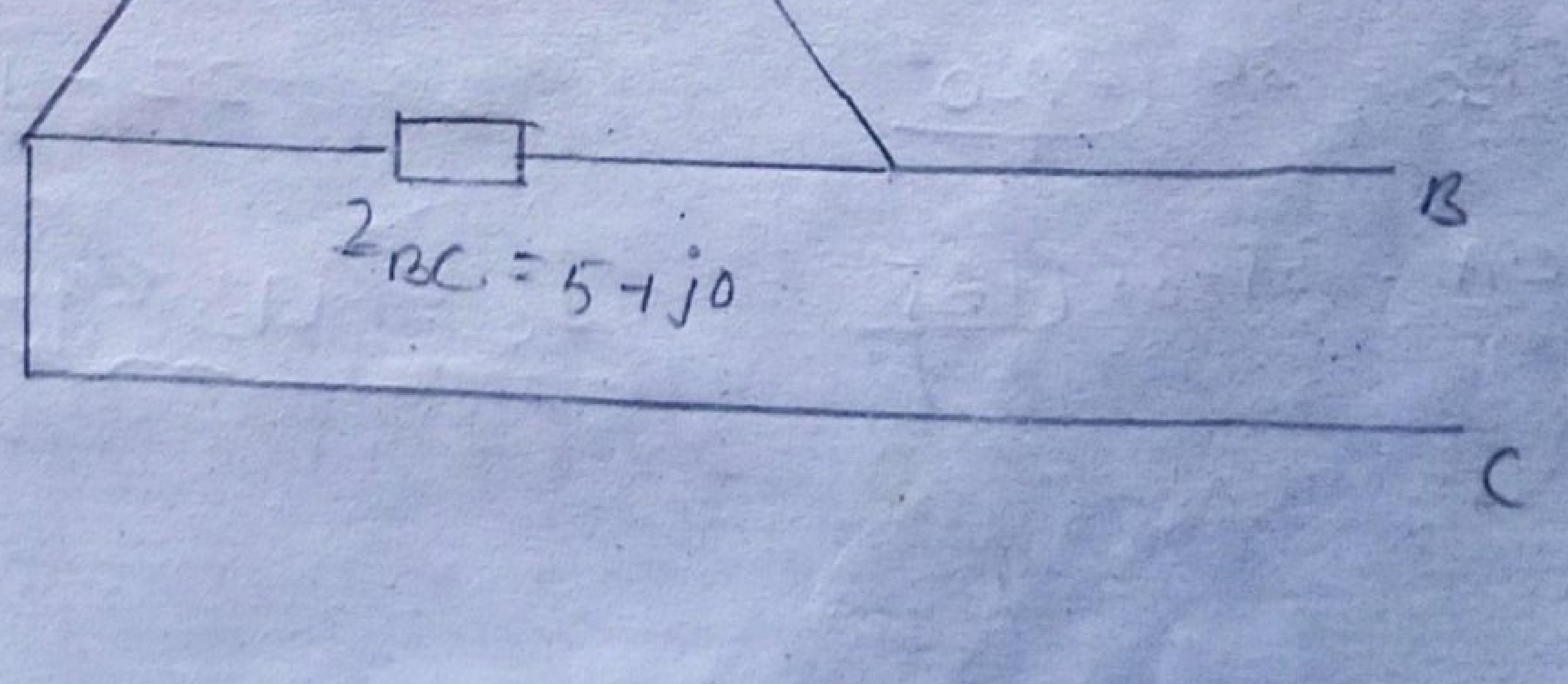
$\therefore I_R = I_L = I_{ph} = 23.09 \angle 0^\circ$

$I_Y = I_L = I_{ph} = 12.75 \angle -153.69^\circ$

$I_B = I_L = I_{ph} = 46 \angle 30^\circ$



pb For the network shown in figure. calculate the line currents and power consum. if the phase sequence is a, b, c



$V_L = 100V$

$Z_{AB} = 3+j4$

$Z_{BC} = 5+j0$

$Z_{CA} = 2-j2$

In delta $V_L = V_{ph}$

$I_L = \sqrt{3} I_{ph}$

$I_L = ?$

$I_A = I_{AB} - I_{CA}$

$I_B = I_{BC} - I_{AB}$

$I_C = I_{CA} - I_{BC}$

$I_{AB} = \frac{V_{AB} \angle 0°}{Z_{AB}}$ $\quad S = V_{AB} I_A + V_{BC} I_B + V_{CA} I_C =$

$= \frac{100 \angle 0°}{3+j4}$ $\quad 12-16i = 20 \angle -0.92$

$= \cancel{33.33 + 4i = 33.33 \angle 0.11}$ $= 20 \angle -53.1$

$I_{BC} = \frac{V_B \angle -120°}{Z_{BC}} = \frac{100 \angle -120°}{5+j10}$ $\quad -10-17.32i$

$= 16.2 - 11.61 = 20 \angle \cancel{-0.61}$ 120

$I_{CA} = \frac{V_{CA} \angle -120°}{Z_{CA}} = \frac{100 \angle -240°}{2-j2}$ $\quad -34.15 + 9.15i$

$= 31.78 - 15.49i = \cancel{35 \angle -0.165}$

$35.35 \angle 165$

$I_A = I_{AB} - I_{CA} = \cancel{12-16i - 31.78 + 15.49i = -19.78 - 0.51i} =$

$= 19.78 \angle -341$

$I_B = I_{BC} - I_{AB} =$

$I_C = I_{CA} - I_{BC} =$

$I_A = 12 - 16i - 34.15 + 9.15i = 22.00 + 1.32i$

$I_A = -0.44 + 0.3525 = 0.57 \angle 141.9 = \cancel{66.09} / 52.2 \angle -28$

$I_B = 0.39 - 0.919 = 1 \angle -66.9$

$I_C = 0.45 - 1.69 = 1.75 \angle -75$

$I_A = 52.2 \angle -28.6$

$I_{AB} = 20 \angle -53.13$ $\quad = I_A = -21.89 - j26.64$

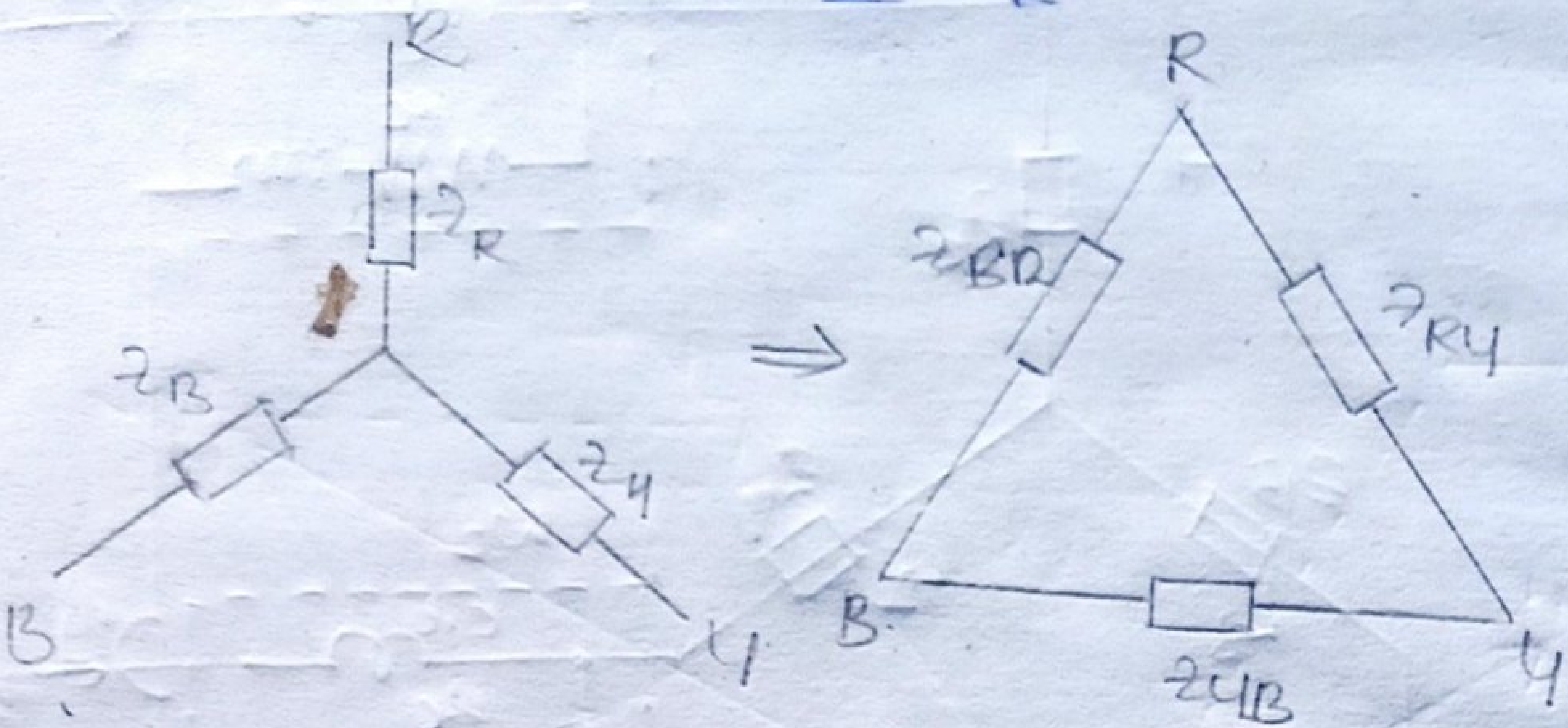
$I_{BC} = 20 \angle -120°$ $\quad I_B = -22 - j1.32$

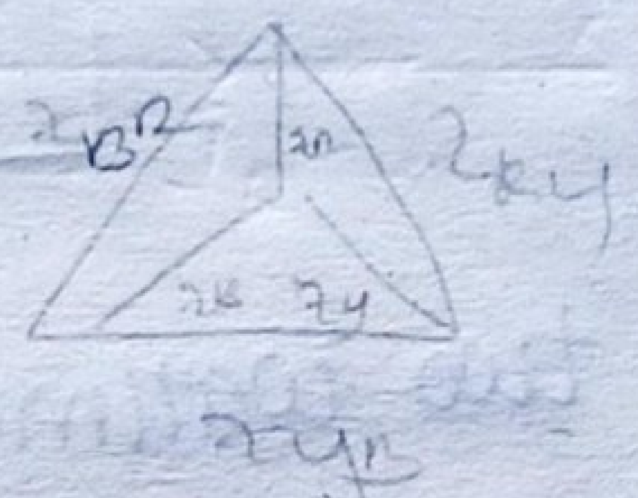
$I_{CA} = 35.35 \angle 165$ $\quad I_C = 43.89 + j27.36$

$S = -2364.19 + j2565.38$

27-09-21

Star to delta transformation:-



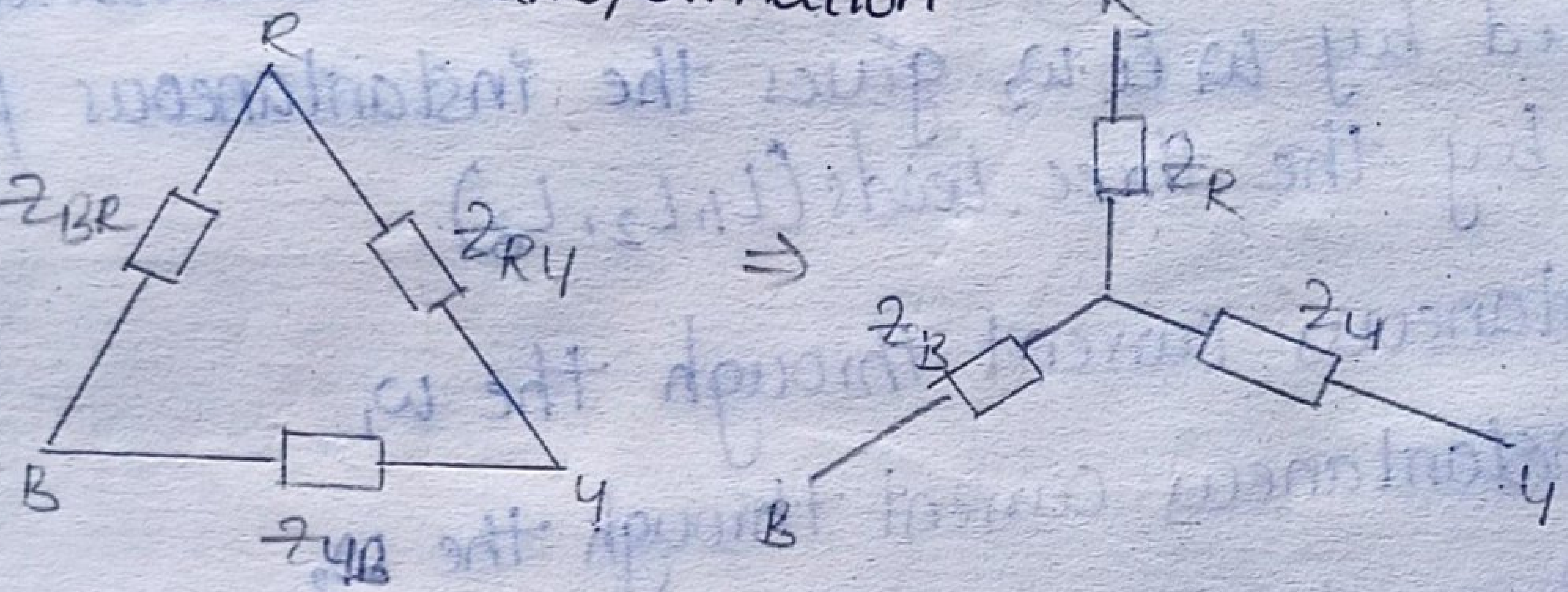


$$Z_{RY} = Z_R + Z_Y + \frac{Z_R Z_Y}{Z_B}$$

$$Z_{YB} = Z_Y + Z_B + \frac{Z_Y Z_B}{Z_R}$$

$$Z_{BR} = Z_B + Z_R + \frac{Z_B Z_R}{Z_Y}$$

Delta to star transformation

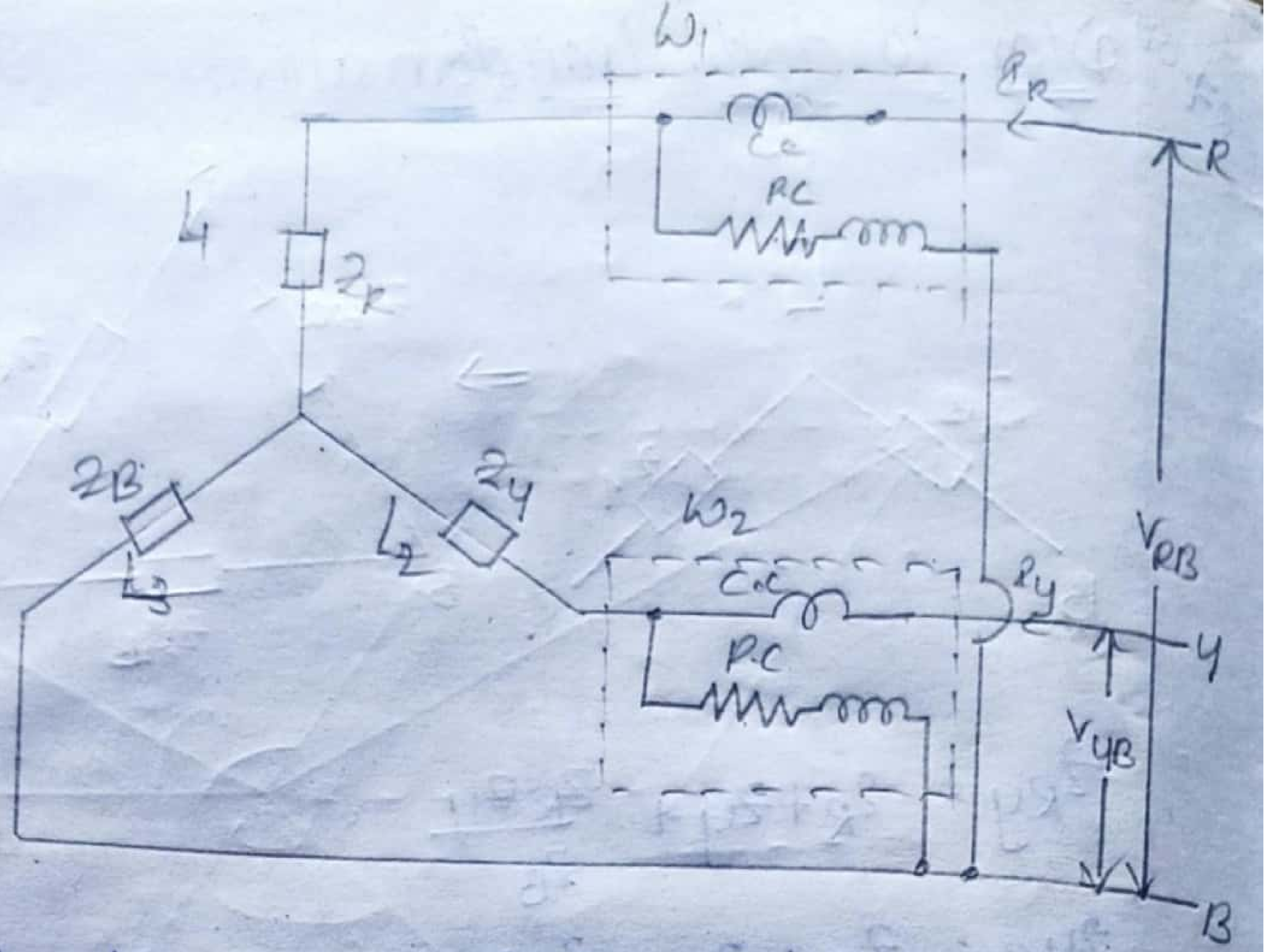


$$Z_R = \frac{Z_{RY} Z_{BR}}{Z_{RY} + Z_{BR} + Z_{YB}}$$

$$Z_Y = \frac{Z_{RY} \cdot Z_{YB}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

$$Z_B = \frac{Z_{YB} \cdot Z_{BR}}{Z_{YB} + Z_{BR} + Z_{RY}}$$

pbr:- 2 Wattmeter method for measurement of 3-ϕ power unbalanced load:-



The two wattmeters are inserted any two lines and pressure coil of each meter is join to the third line in which the current coil will not included

It can prove that the sum of the instantaneous power indicated by W_1 & W_2 gives the instantaneous power absorb by the three loads (L_1, L_2, L_3)

i_R = Instantaneous current through the W_1

i_Y = Instantaneous current through the W_2

V_{RB} = Instantaneous potential difference accross W_1

V_{YB} = Instantaneous potential difference across W_2

Instantaneous power measured by $W_1 = V_{RB} i_R$

$$W_1 = (V_R - V_B)\, i_R$$

Instantaneous power measured by $W_2 = V_{YB} i_Y$

$$W_2 = (V_Y - V_B)\, i_Y$$

Total power measured by Two wattmeters

$= W_1 + W_2$

$= V_R i_R - V_B i_R + V_Y i_Y - V_B i_Y$ $\quad (\because I_R + i_Y + i_B = 0)$

$= V_R i_R + V_Y i_Y - (V_B (i_R + i_Y))$

$= V_R i_R + V_Y i_Y - V_B(-i_B)$

$= V_R I_R + V_Y I_Y + V_B I_B$

28-09-22.

UNIT - II.

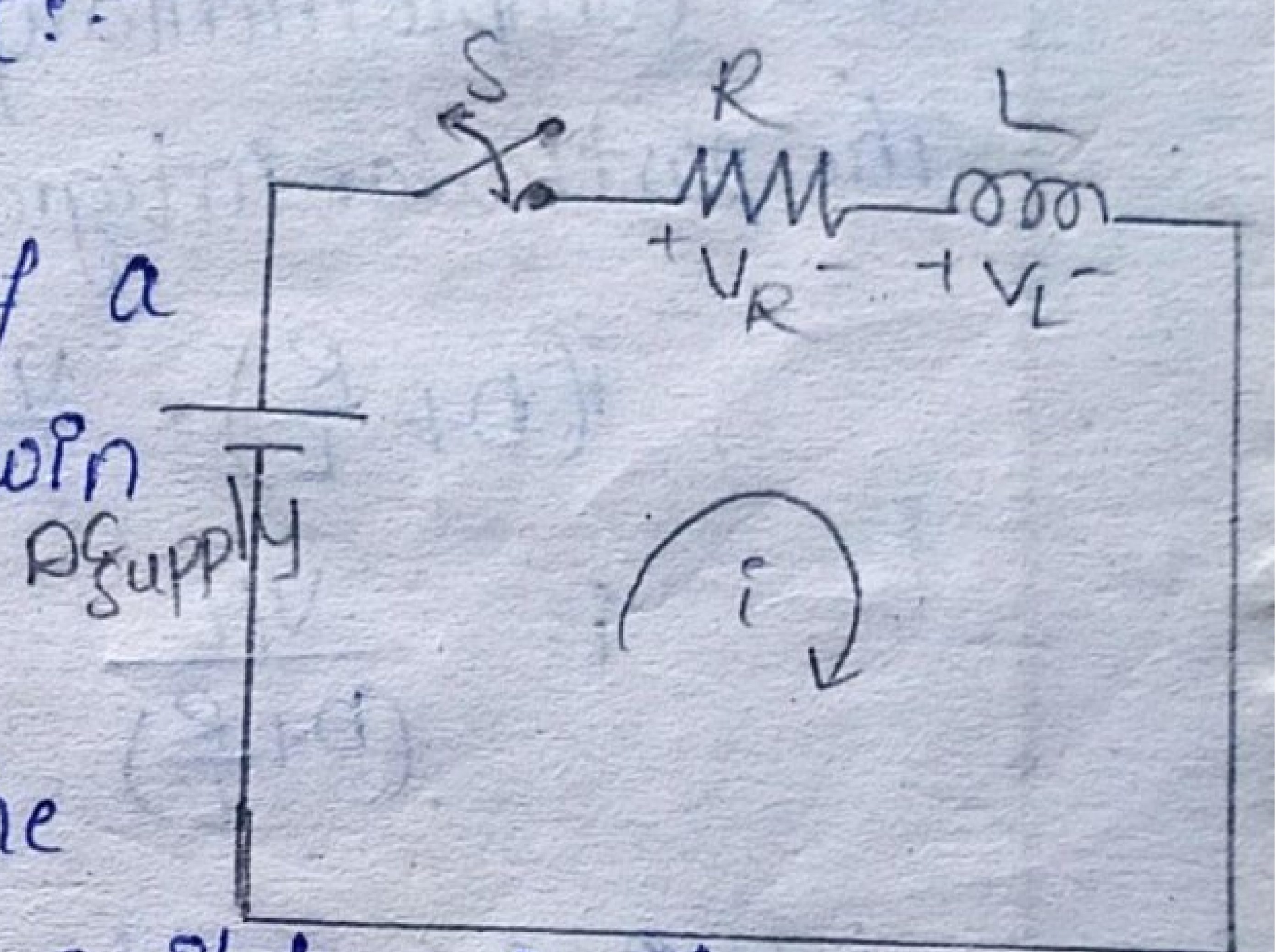
DC TRANSIENTS

Transients:-

In any transmission line the voltage is fluctuating (or) inbalanced due to short circuiting (or) if any fault (or) any inbalances is called Transients Voltage

Transients is calculated for circuit elements
i, R-L circuit   ii, R-C circuit   iii, R-L-C circuit

DC response of an R-L circuit:-



Consider a circuit consisting of a resistance & inductance is shown in figure.

Initially the inductor in the circuit is uncharged. When the switch S is closed we can find the complete solution for the current.

apply KVL in above circuit

$V = V_R + V_L$

$V = iR + L\frac{di}{dt}$.

* The above eqn i is the current flowing through the circuit & V is the applied constant voltage.
* The applied voltage V is applied to the circuit when the switch S is closed.
* The above eqn is linear differential of a first order.

$V = iR + L\frac{di}{dt}$

$\frac{V}{L} = \frac{iR}{L} + \frac{di}{dt}$

$$\frac{di}{dt} + i\frac{R}{L} = \frac{V}{L}$$

The complementary function is

$\frac{di}{dt} = D$ as operator

$$Di + i\frac{R}{L} = \frac{V}{L}$$

$$i\left(D + \frac{R}{L}\right) = \frac{V}{L}$$

Solution

the complementary function $C.F = ke^{Dt}$.

the C.F $i\left(D+\frac{R}{L}\right) = 0$

$$D + \frac{R}{L} = 0$$

$$D = \frac{-R}{L}$$

The complementary function $C.F = ke^{-\frac{R}{L}t}$.

The particular integral

$$i\left(D + \frac{R}{L}\right) = \frac{V}{L}$$

$$i = \frac{V/L}{\left(D + \frac{R}{L}\right)}$$

$$P.I. i = \frac{\frac{V}{L}}{\left(D + \frac{R}{L}\right)}$$

$$i = \frac{\frac{V}{L}}{\frac{R}{L}\left(1 + \frac{D}{R/L}\right)}$$

$$i = \frac{V}{R}\left(1 + \frac{D}{R/L}\right)^{-1}$$

$$i = \frac{V}{R}(1)$$

$$i = \frac{V}{R}$$

The complete Solution $i = C.F + P.I$

$$i = ke^{-\frac{R}{L}t} + \frac{V}{R}$$

Becaus of the presence of the inductance doesn't allow the sudden change in current i-e

$t = 0$; when $i = 0$.

$$0 = ke^{-\frac{R}{L}(0)} + \frac{V}{R}$$

$$0 = ke^{0} + \frac{V}{R}$$

$$0 = k + \frac{V}{R}$$

$$k = \frac{-V}{R}$$

The complete solution/final solution is

$$i = \frac{-V}{R}e^{-\frac{R}{L}t} + \frac{V}{R} = \frac{V}{R} - \frac{V}{R}e^{-(R/L)t}$$

$$i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right).$$

The above equation consisting of two parts.

i. The study part is $\frac{V}{R}$

ii. The transient part is $\frac{V}{R}e^{\left(-\frac{R}{L}\right)t}$.

voltage across the resistor $V_R = iR$

$$V_R = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)R$$

$$V_R = V\left(1 - e^{\left(-\frac{R}{L}\right)t}\right)$$

voltage across the inductor $V_L = L\frac{di}{dt}$

$$V_L = L\frac{d}{dt}\left(\frac{V}{R}\left(1 - e^{\left(-R/L\right)t}\right)\right)$$

$$V_L = L\frac{V}{R}\frac{d}{dt}\left(1 - e^{\left(-\frac{R}{L}\right)t}\right)$$

$$= \frac{LV}{R}\left(-e^{\left(-\frac{R}{L}\right)t}\right)\left(\frac{-R}{L}\right)$$

$$= Ve^{\left(-\frac{R}{L}\right)t}.$$

power flowing through resistor

$$P_R = V_R i$$

$$= V\left[1 - e^{\left(-\frac{R}{L}\right)t}\right]\left[\frac{V}{R}\left(1 - e^{\left(-\frac{R}{L}\right)t}\right)\right]$$

$$= \frac{V^2}{R}\left(1 - e^{\left(-\frac{R}{L}\right)t}\right)^2$$

power flowing through inductor

$$P_L = V_L i$$

$= V e^{(-\frac{R}{L})t} \left[\frac{V}{R}(1-e^{(-R/L)t}\right]$

$- \frac{V^2}{R}\left[e^{(-\frac{R}{L})t}(1-e^{(-\frac{R}{L})t})\right]$

9-09-22 that are $\frac{L}{R}$ is called time constant and it is denoted τ

$\tau = \frac{L}{R}$ is known time constant of the circuit and is defined as the interval after which current or voltage changes 63.2% of its total change

At $t=0$

$i = \frac{V}{R}(1-e^{-\frac{R}{L}t})$

$= \frac{V}{R}(1-1)$

$= 0$

$V_R = V(1-e^{-\frac{R}{L}t})$

$= V(1-1)$

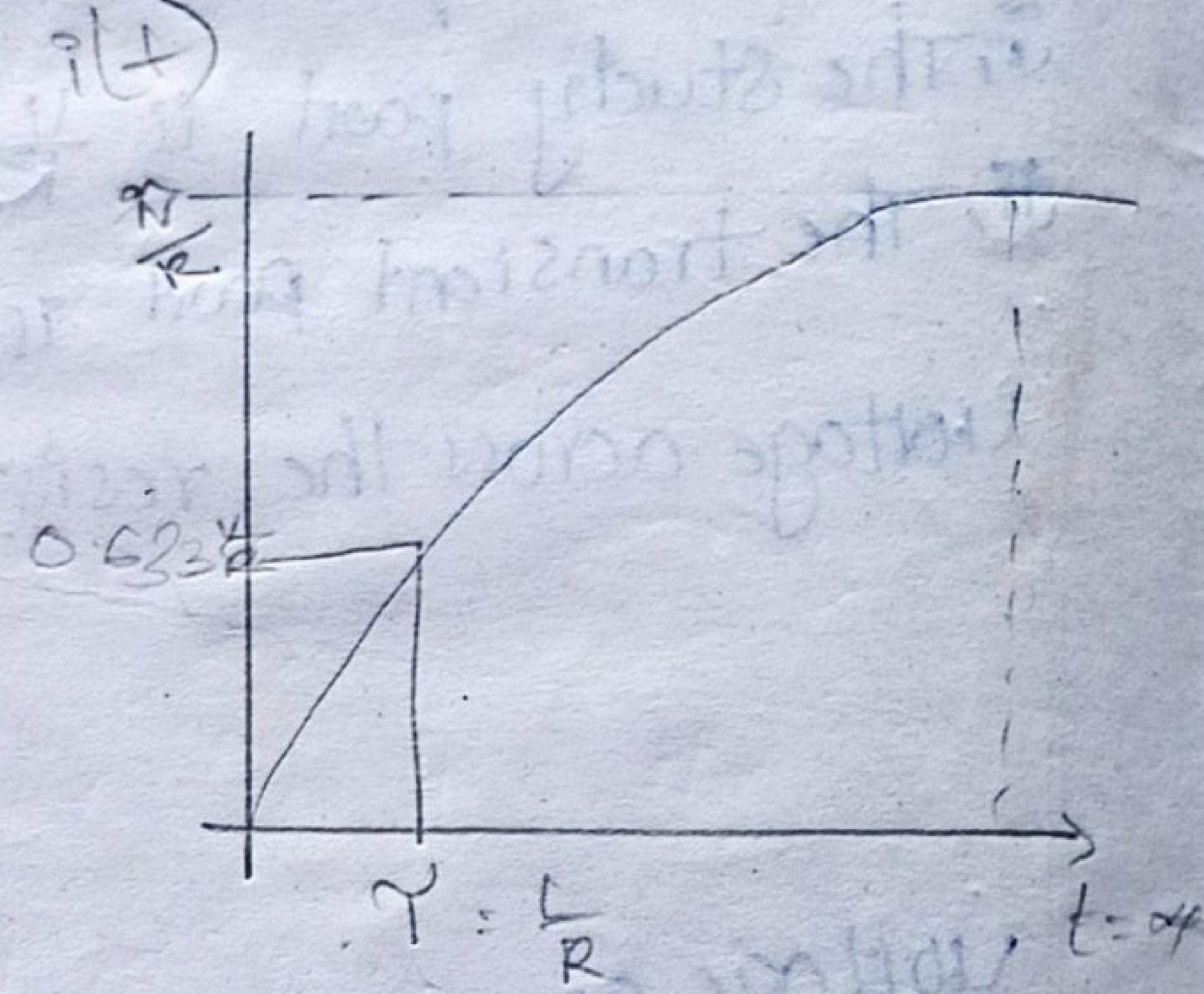
$= 0$

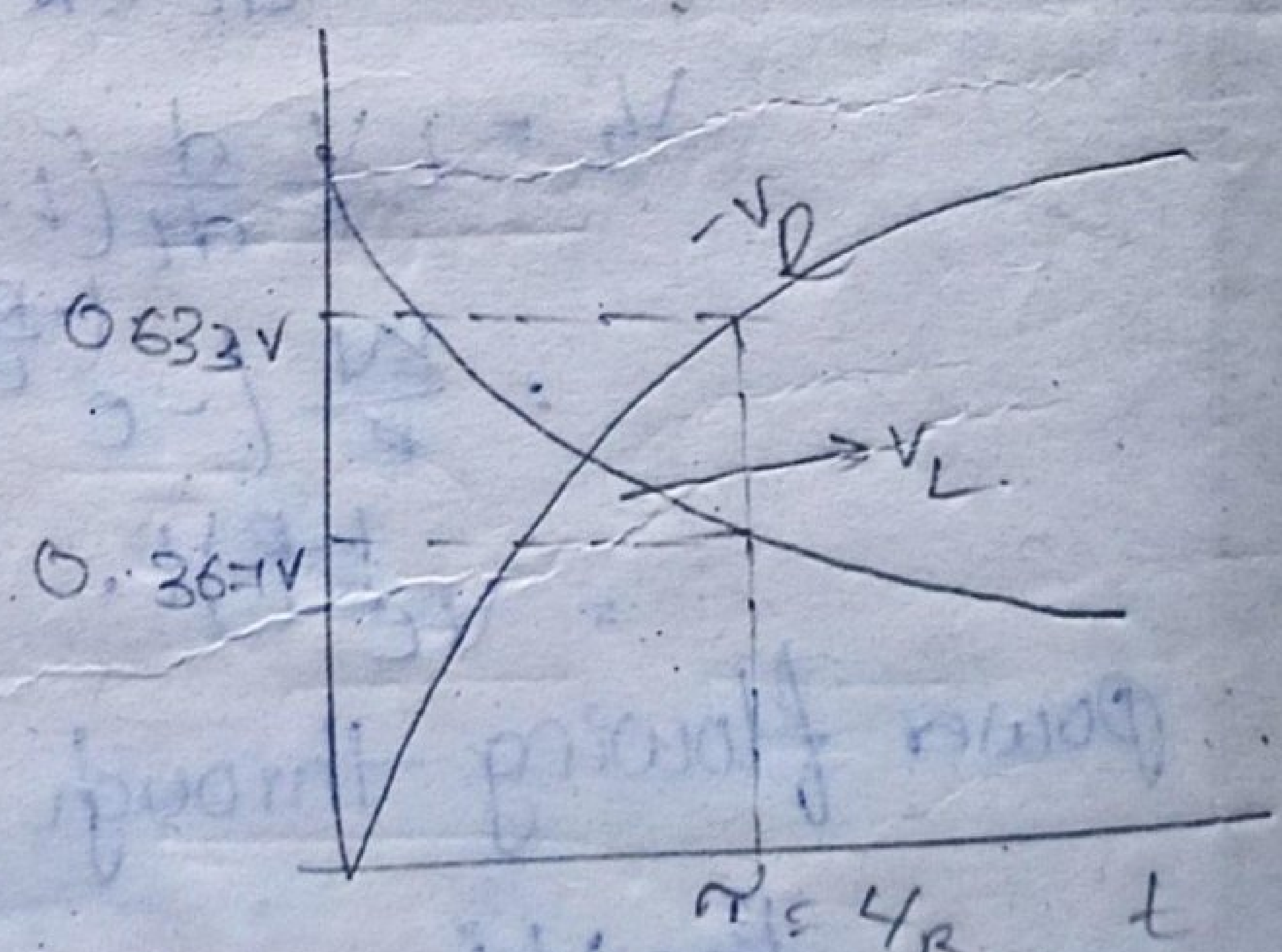
$V_L = V(e^{-R/L t})$

$= V(1)$

$= V$

L acts ~~short~~ open circuit





At $t=\infty$

$i = \frac{V}{R}(1-e^{-\frac{R}{L}\infty})$

$= \frac{V}{R}$

$V_R = V(1-e^{-R/L\infty})$

$= V$

$V_L = V e^{(-R/L)\infty}$

$= 0 \longrightarrow$ it acts as short circuit

At $t = \tau = \frac{L}{R}$

$i = \frac{V}{R}(1-e^{(-\frac{R}{L})\frac{L}{R}})$

$$i = \frac{V}{R}(1 - e^{-1}) = \frac{V}{R}\left(1 - \frac{1}{e}\right)$$

$$i = \frac{V}{R}(1 - 0.367)$$

$$i = \frac{V}{R}(0.633)$$

$$V_R = V(1 - e^{-\frac{R}{L}\frac{L}{R}})$$

$$= V(1 - e^{-1})$$

$$V(0.633)$$

$$V_L = V(e^{-\frac{R}{L}\frac{L}{R}})$$

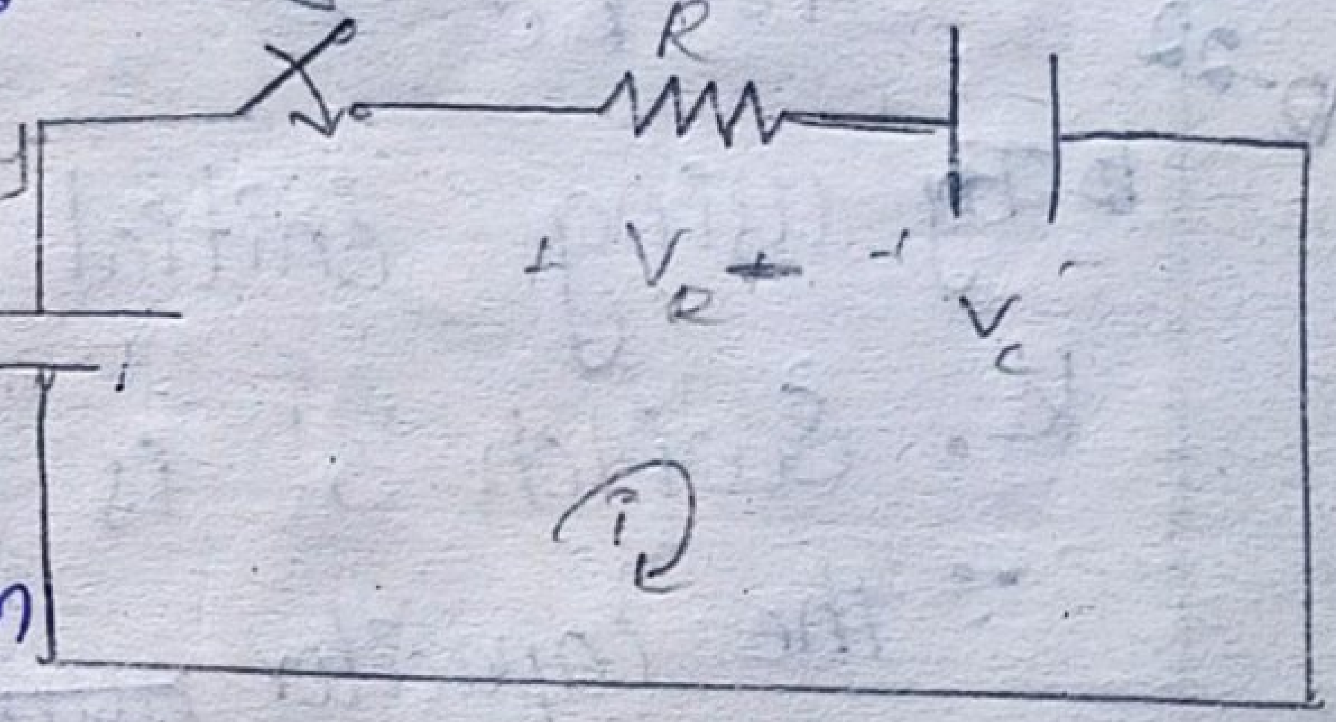
$$= V(e^{-1})$$

$$= V(0.367)$$

D.C. Response of a R-C circuit:-

Consider a circuit consisting of resistance in series with capacitance as shown in figure

When the switch S is closed at $t=0$. we can determine the complete solution for the current. applying KVL



$$V = V_R + V_C$$

$$V = iR + \frac{1}{C}\int i\,dt$$

differentiating above Equation

$$0 = R\frac{di}{dt} + \frac{1}{C}i$$

$$0 = \frac{di}{dt} + \frac{1}{RC}i$$

The complementary function C.F is

$$0 = Di + \frac{1}{RC}i$$

$$i\left(D + \frac{1}{RC}\right) = 0$$

The $C.F = Ke^{Dt}$.

$$i\left(D + \frac{1}{RC}\right) = 0$$

$D + \frac{1}{RC} = 0$

$D = \frac{-1}{RC}$

Solutional
Then complementary function $= ke^{\frac{-1}{RC}t}$

The particular integral p.i is

$i\left(D + \frac{1}{RC}\right) = 0$

$i = \frac{0}{D + \frac{1}{RC}}$

$i = 0$

The Complete Solution $i = c.f + p.i$

$i = ke^{-\frac{1}{RC}t} + 0$

$i = ke^{\left(\frac{-1}{RC}\right)t}.$

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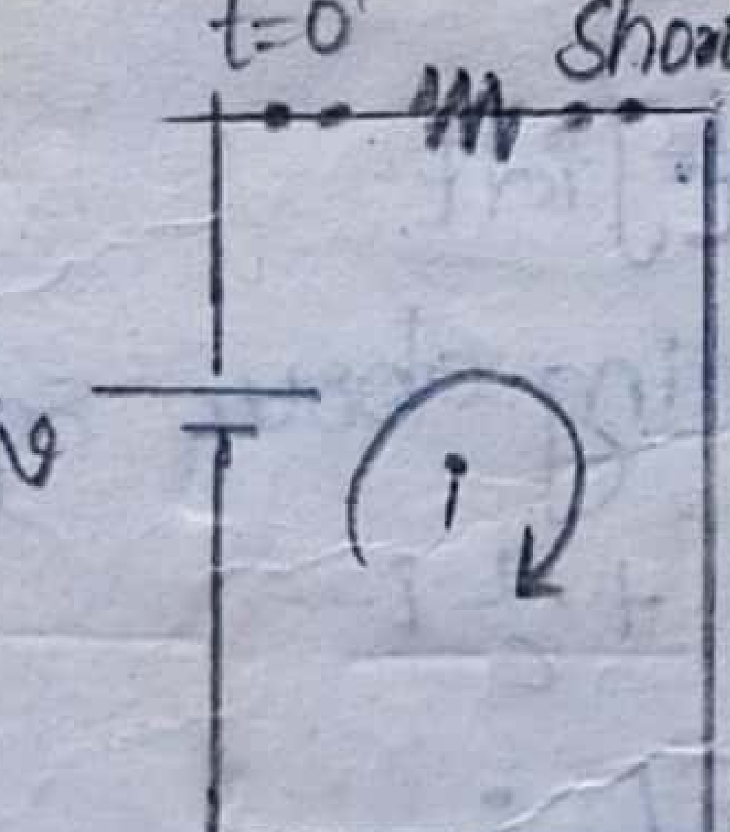
By using initial condition we find the value of k. Switch 's' is closed at $t=0$.

$\because$ The capacitor ~~num~~ never allows the sudden changes in voltage. it will act as a short circuit at $t=0+$



At $t=0+$

$i = \frac{V}{R}$

$i = ke^{\left(\frac{-1}{RC}\right)t}$

$\frac{V}{R} = ke^{\left(\frac{-1}{RC}\right)(0)}$

$\boxed{\frac{V}{R} = k}$

The final solution is

$i = \frac{V}{R} e^{\left(\frac{-1}{RC}\right)t}$

voltage across the resistance

$V_R = iR$

$V_R = \frac{V}{\cancel{R}} e^{\left(-\frac{1}{RC}\right)t} \cancel{R}$ $\quad$ $t=0$.

$V_R = Ve^{\left(-\frac{1}{RC}\right)t}$. $\quad$ $V_R = Ve^{\left(-\frac{1}{RC}\right)0}$

$V_R = V$

voltage across capacitor

$$V_C = \frac{1}{C}\int i\,dt$$

$$V_C = \frac{1}{C}\int \frac{V}{R} e^{\left(-\frac{1}{RC}\right)t} dt$$

$$V_C = \frac{V}{CR}\int e^{\left(-\frac{1}{RC}\right)t} dt$$

$$V_C = \frac{V}{\cancel{RC}} \frac{e^{\left(-\frac{1}{RC}\right)t}}{-\frac{1}{RC}} + k$$

$$V_C = -Ve^{\left(-\frac{1}{RC}\right)t} + k$$

at $t=0$,

$$V_C = -Ve^{\left(-\frac{1}{RC}\right)0} + k$$

$$V_C = -V + k$$

at $V_C = 0$

$$0 = -V + k$$

$$\boxed{k = V}$$

At $t=0$ the voltage across the capacitor is 0 $(\because V_C = 0)$

$$V_C = -Ve^{\left(-\frac{1}{RC}\right)t} + V$$

$$V_C = V\left(1 - e^{\left(-\frac{1}{RC}\right)t}\right)$$

The above equation is the final solution

power through resistor $P_R = V_R\, i$

$$P_R = Ve^{\left(-\frac{1}{RC}\right)t} \frac{V}{R} e^{\left(-\frac{1}{RC}\right)t}$$

$$= \frac{V^2}{R}\left(e^{\left(-\frac{1}{RC}\right)t}\right)^2$$

power through capacitor

$$P_C = V_C i$$

$$= V\left(1-e^{\left(\frac{-1}{RC}\right)t}\right)\frac{V}{R}e^{\left(\frac{-1}{RC}\right)t}$$

$$= \frac{V^2}{R}\left[e^{\left(\frac{-1}{RC}\right)t} - \left(e^{\left(\frac{-1}{RC}\right)t}\right)^2\right]$$

When Switch S is closed the response decays with time as shown in figure.

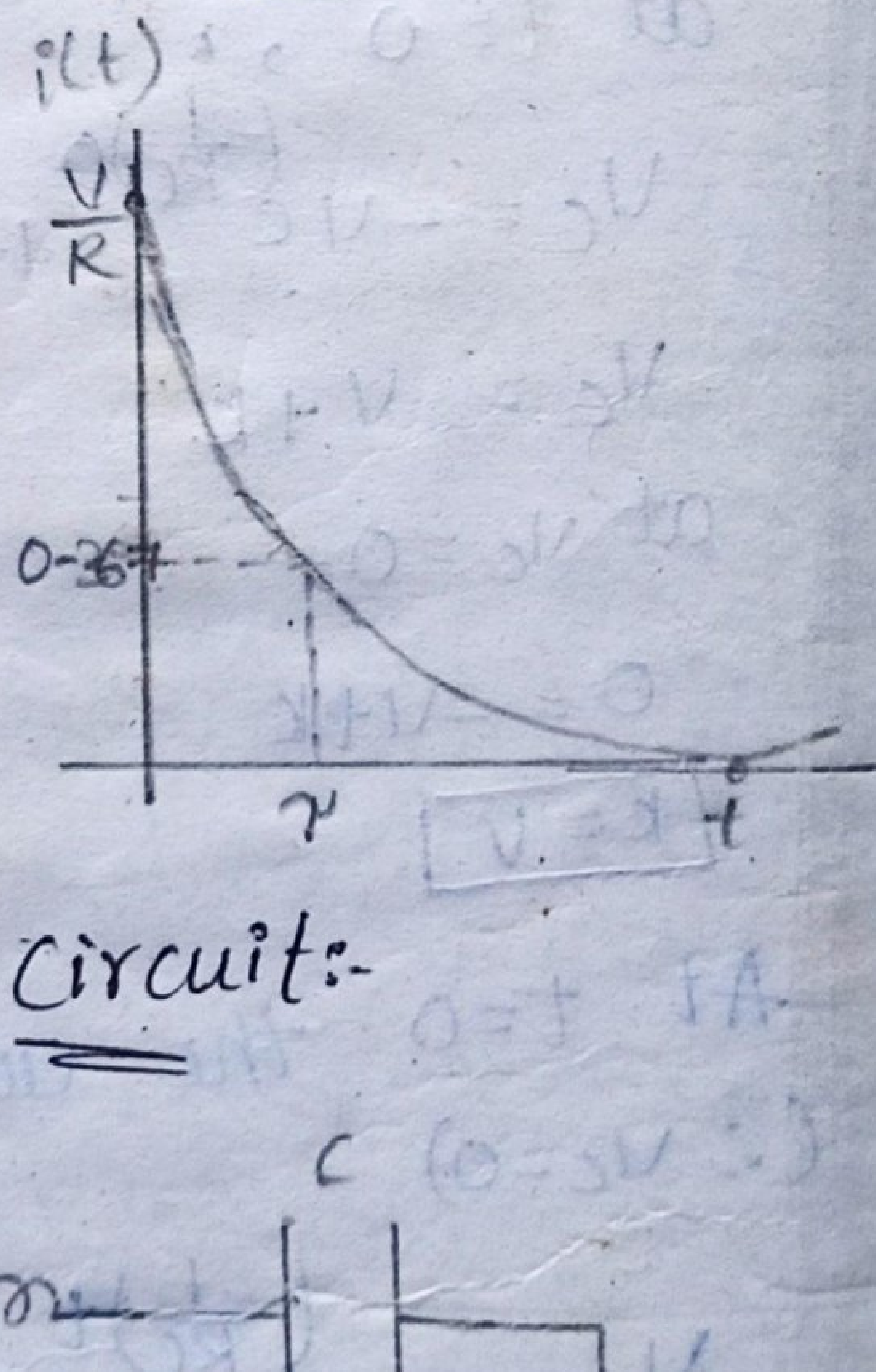
The Quantity RC is the time constant and it denoted by τ

$\underline{\tau = RC}$ ⟶ Time constant in sec.

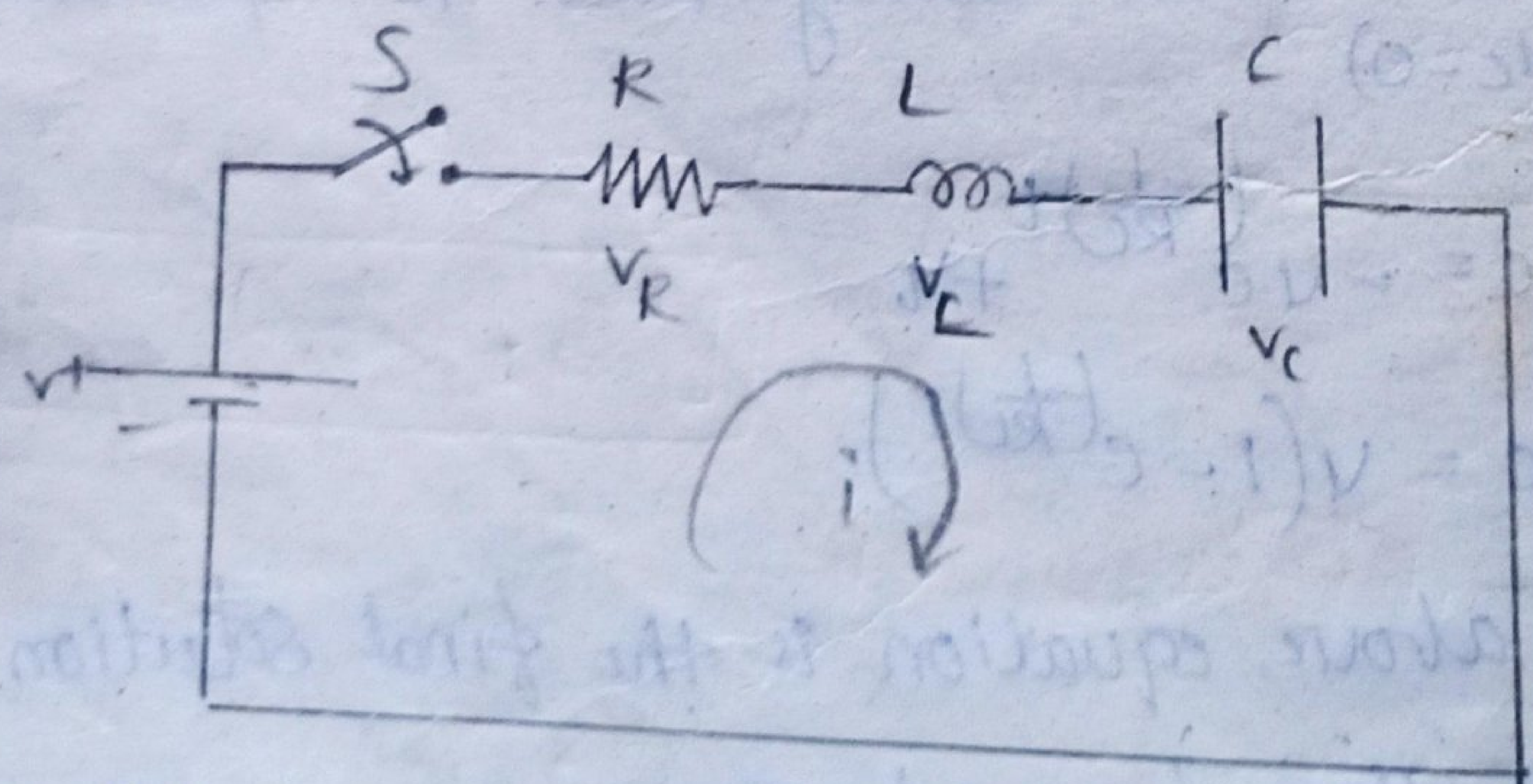
$t=0 \rightarrow i = \frac{V}{R}$

$t=\infty \Rightarrow i = 0$

$t = \tau \Rightarrow i = \frac{V}{R}(0.367)$



DC Response of a R-L-C circuit:-



consider a circuit consisting of a resistance inductance, capacitance as shown in figure initially the capacitor & inductor are un charge in series with resistor.

When switch S is closed at $t=0$. we can find the complete solution (of the) for the current. applying KVL in above circuit.

$$-iR - L\frac{di}{dt} - \frac{1}{C}\int i\,dt + V = 0$$

$$V = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

defferentiating above equation for complete sol

$$0 = R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{1}{C}i$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$$

$\frac{d}{dt} \Rightarrow D$ as operator.

$$i\left[D^2 + \frac{R}{L}D + \frac{1}{LC}\right] = 0$$

The roots of the above eq is α, β

$$\underset{D_1, D_2}{\alpha, \beta} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\underset{D_1, D_2}{\alpha, \beta} = \frac{-\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - 4(1)\frac{1}{LC}}}{2(1)}$$

$$\underset{D_1, D_2}{\alpha, \beta} = \frac{-\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - \frac{4}{LC}}}{2} \qquad \boxed{\frac{R^2}{L^2}\left(1 - \frac{4L}{RC}\right)}$$

$$\underset{D_1, D_2}{\alpha, \beta} = \frac{-\frac{R}{L} \pm \frac{R}{L}\sqrt{1 - \frac{4L}{RC}}}{2}$$

$$\underset{D_1, D_2}{\alpha, \beta} = \frac{\frac{R}{L}\left(-1 \pm \sqrt{1 - \frac{4L}{RC}}\right)}{2}$$

$$D_1, D_2 = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{\cancel{4}}{4LC}}$$

Now it is in the form $\alpha \pm \beta$

$$\therefore \alpha = \frac{-R}{2L} \qquad \beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

The current response

$$i = k_1 e^{D_1 t} + k_2 e^{D_2 t}$$

Where k_1, k_2 are the constants

D_1, D_2 are the roots

case i, when

$$\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$$

At this time β is positive real Quantity and +ve (positive real quantity)

Hence the roots are D_1, D_2 are real and unequal

$$D_1 = \alpha + \beta \qquad D_2 = \alpha - \beta$$

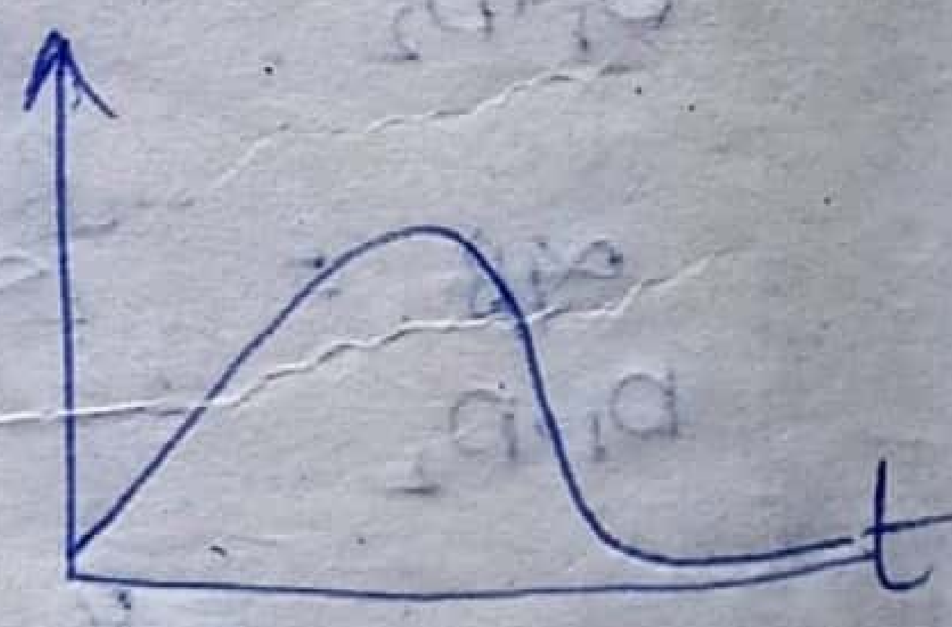
The current solution is

$$i = k_1 e^{D_1 t} + k_2 e^{D_2 t}$$

$$i = k_1 e^{(\alpha+\beta)t} + k_2 e^{(\alpha-\beta)t}$$

$$i = k_1 e^{\alpha t} e^{\beta t} + k_2 e^{\alpha t} e^{-\beta t}$$

$$\boxed{i = e^{\alpha t}\left[k_1 e^{\beta t} + k_2 e^{-\beta t}\right]}$$



The above equation is the current solution

case ii, when $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$

At this time β is imaginary quantity & the roots D_1, D_2 are complex conjugate

$D_1 = \alpha + j\beta \;,\; D_2 = \alpha - j\beta$

$i = k_1 e^{D_1 t} + k_2 e^{D_2 t}$

$i = k_1 e^{(\alpha + j\beta)t} + k_2 e^{(\alpha - j\beta)t}$

$i = k_1 e^{\alpha t} \cdot e^{j\beta t} + k_2 e^{\alpha t} e^{-j\beta t}$

$i = e^{\alpha t}\left[k_1 e^{j\beta t} + k_2 e^{-j\beta t}\right]$

The Sd current solution is oscillatory and underdamped

case iii, $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$

~~xice~~ $D_1 = \alpha + j\beta \;,\; D_2 = \alpha - j\beta$

At this time β is zero Hence D_1, D_2 are real & equal

$\therefore D_1 = D_2 = \lambda$.

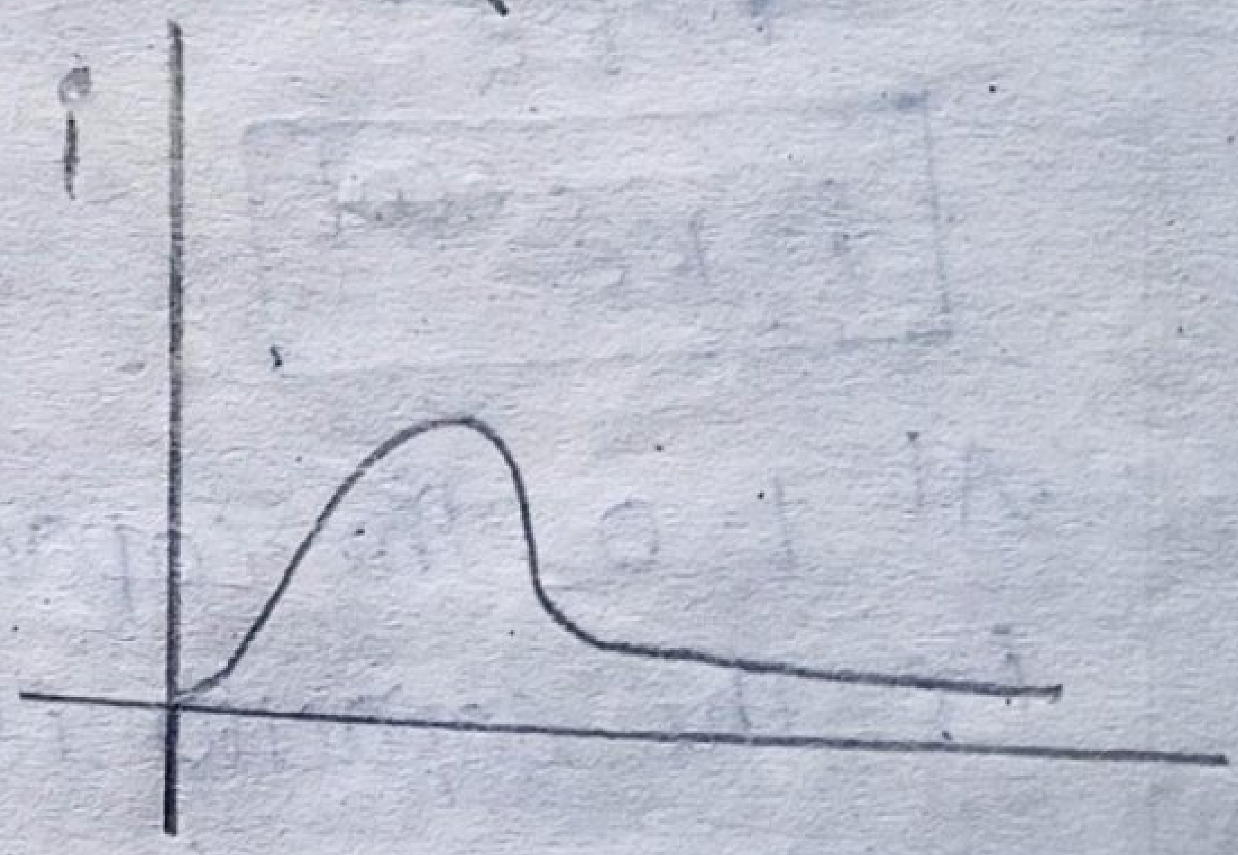
$D_1 = \alpha + 0 \qquad D_2 = \alpha - 0$

$D_1 = \alpha \qquad D_2 = \alpha$

$i = k_1 e^{D_1 t} + k_2 e^{D_2 t}$

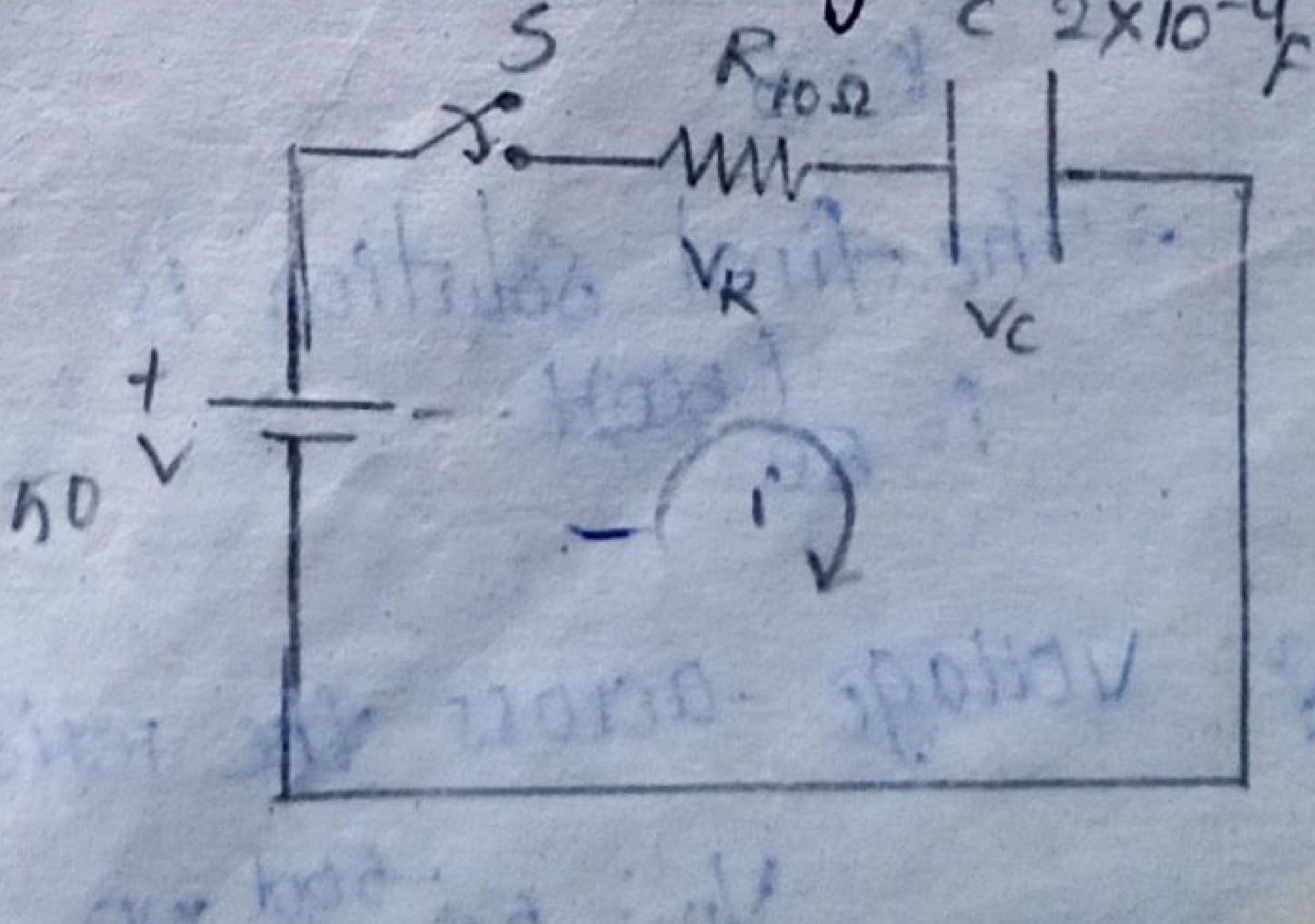
$i = k_1 e^{\alpha t} + k_2 e^{\alpha t}$

$i = e^{\alpha t}[k_1 + k_2]$



The current Solution is critically damped

Pb:1 For the circuit shown in figure determined the current at any time $t > 0$. The switch S is closed at $t=0$ assume initial current & initial charge on the capacitor is zero



Applying the kvl

$V = V_R + V_C$

$50 = i(10) + \frac{1}{2\times10^{-4}}\int i\,dt$

$50 = 10i + \frac{1}{2\times10^{-4}}\int i\,dt$

differentiating above equation

$0 = 10\frac{di}{dt} + \frac{1}{2\times10^{-4}} i$

$0 = \frac{di}{dt} + \frac{1}{2\times10^{-4}\times10} i$

$0 = \frac{di}{dt} + \frac{1}{2\times10^{-3}} i$

$0 = Di + \frac{1}{2\times10^{-3}} i$

$0 = Di + (0.5)\times10^{3} i$

$0 = i(D + (0.5)\times10^{3})$

$0 = i(D+500)$ $\Rightarrow D+500=0$ $D=-500$

The 8 current solution is

$i = ke^{Dt}$

$\boxed{i = ke^{-500t}}$

At $t=0$ the capacitor never allows sudden changes i.e the capacitor is short ckt.

$\therefore V_C = 0$

$i = \frac{V}{R} = \frac{50}{10}$

$i = 5A$

From $i = ke^{-500t}$

$5 = ke^{(-500)0}$

$k = 5$

$\therefore$ The final solution is

$i = 5e^{(-500)t}$

Voltage across the resistor $V_R = iR$

$V_R = 5e^{-500t} \times 10$

$V_R = 50$

voltage across the capacitor $V_C = \frac{1}{C}\int i dt$

$$V_C = \frac{1}{2\times10^{-4}}\int 5 e^{-500t} dt$$

$$= \frac{5}{2\times10^{-4}}\int e^{(-500t)} dt$$

$$= \frac{\cancel{5}}{2\times10^{-4}} \frac{e^{-500t}}{-\cancel{500}_{100}}$$

$$= \frac{-1}{2\times10^{-2}} e^{-500t}$$

$$= -(0.5)\times10^{2} e^{-500t}$$

$$= \cancel{50} \times e^{-500t}$$

$$= -50 \times e^{-500t}$$

pb:2 A Series R-L circuit has $R = 25\Omega$ & $L = 5H$. A dc voltage of 100 volts is applied at $t=0$.

i, Find the equation for charging current, voltage across R & L

ii, The current in the circuit 0.5 sec later

iii, The time at which the drops across R & L are same

i charging current $i = \frac{V}{R}(1 - e^{-\frac{R}{L}t})$

$$i = \frac{100}{25}(1 - e^{-\frac{25}{5}t})$$

$$= 4(1 - e^{-5t})$$

$$\cancel{=} 4(1 - e^{\infty(-5)})$$

$$\cancel{=} 4(1-0)$$

$$\cancel{=} 4$$

voltage across ressistance $V_R = iR$

$$V_R = 4(1 - e^{-5t})(25)$$

$$V_R = 100(1 - e^{-5t})$$

voltage across inductance $V_L = \frac{1}{6} L \frac{di}{dt}$

$V_L = 5 \frac{d}{dt}(4(1-e^{-5t}))$

$= 20 \frac{d}{dt}(1-e^{-5t})$

$= 20(-e^{-5t}(-5))$

$= 100 e^{-5t}$

At $t = 0.5$ sec

$i = 4(1-e^{-5t})$

$i = 4(1-e^{-5(0.5)})$

$i = 4(1-e^{-2.5})$

$i = 4(1-0.08)$

$i = 4(0.92)$

~~$i = 0.368$ amp.~~

$i = 3.68$ amp

iii) To satisfying the condition $V_R = V_L$

3/10/22 At which time

$V_R = V_L$ at supply voltage $= 100V$

$V_R = V_L = 50$

$iR = L\frac{dI}{dt}$

$\cancel{50} = \cancel{100}\, 2 e^{-5t}$

$e^{-5t} = \frac{1}{2}$

$-5t = \log(\frac{1}{2})$

$t = \frac{+1}{5} \log(\frac{1}{2})$

$t = 0.060$

Laplace transforms:

$L(f(t)) = f(s)$

$L(1) = \frac{1}{S}$

$L(K) = \frac{K}{S}$

$L(t) = \frac{1}{S^2}$.

$L[e^{-at}] = \frac{1}{S+a}$

$L[f'(t)] = Sf(S) - f(0)$

$L[f''(t)] = S^2 f(s) - Sf(0) - f'(0)$

Laplace of integration

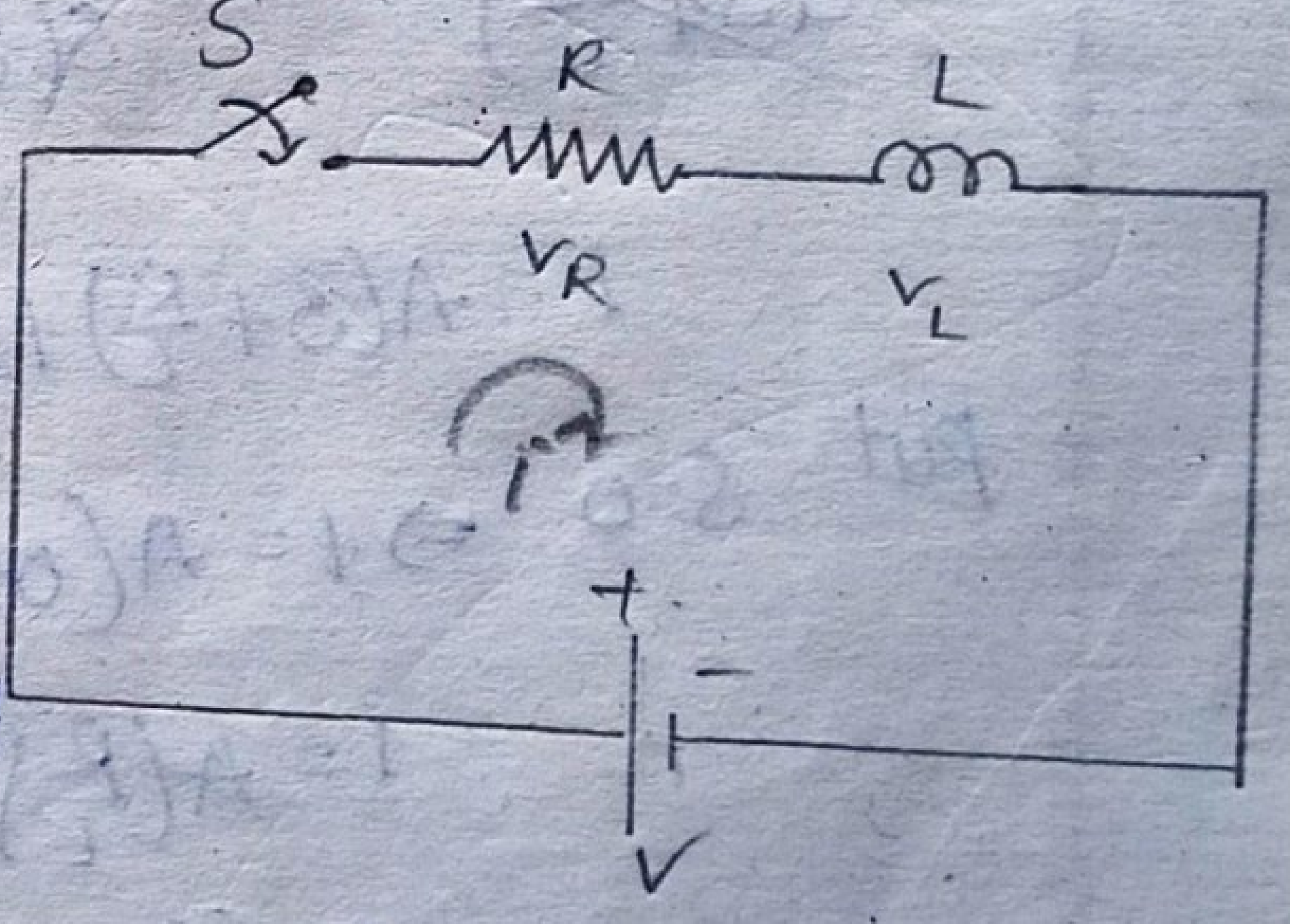
$L[\int_0^t f(t)] = \frac{F(s)}{S}$

$L[\cos\omega t] = \frac{S}{S^2+\omega^2}$

$L[\sin\omega t] = \frac{\omega}{S^2+\omega^2}$

Transient response of R-L circuit by using laplace transform:-

A circuit consisting of a Resistance in series with inductance as shown in fig



Apply the KVL in above ckt

$V = iR + L\frac{di}{dt}$

Apply laplace transform

$V = i(t)R + L\frac{dI(t)}{dt}$

$\frac{V}{S} = RI(S) + L[SI(S) - I(0)]$

before closing the switch $I(0) = 0$

$\frac{V}{S} = RI(S) + LSI(S)$

$\frac{V}{S} = I(s)[R+LS]$

$I(s) = \frac{V/S}{R+LS}$

$= \frac{V}{S(R+LS)}$

$= \frac{V}{LS(S+\frac{R}{L})}$

$I(s) = \frac{V/L}{S(S+\frac{R}{L})}$

$I(S) = \frac{V}{L}\left(\frac{1}{S(S+\frac{R}{L})}\right)$

Apply partial differention

$I(S) = \frac{V}{L}\left(\frac{A}{S} + \frac{B}{S+R/L}\right) \longrightarrow (1)$

$\frac{1}{S(S+\frac{R}{L})} = \frac{A}{S} + \frac{B}{S+R/L}$

$\frac{1}{\cancel{S(S+\frac{R}{L})}} = \frac{A(S+R/L) + BS}{\cancel{S(S+\frac{R}{L})}}$

$1 = A(S+\frac{R}{L}) + BS$

put $S=0 \Rightarrow 1 = A(0+\frac{R}{L}) + B(0)$

$1 = A(\frac{R}{L})$

$A = \frac{L}{R}$

put $S = \frac{-R}{L} \Rightarrow 1 = A\left[\frac{-R}{L} + \frac{R}{L}\right] + B\left[\frac{-R}{L}\right]$

$1 = A(0) + B(\frac{-R}{L})$

$1 = B(\frac{-R}{L})$

$B = \frac{-L}{R}$

A&B values sub in eq ①

$$\frac{A}{S} + \frac{B}{S+\frac{R}{L}} = \frac{\frac{L}{R}}{S} + \frac{-\frac{L}{R}}{S+\frac{R}{L}}$$

$$= \frac{L/R}{S} - \frac{L/R}{S+\frac{R}{L}}$$

$$I(s) = \frac{V}{L}\left(\frac{1}{S(S+\frac{R}{L})}\right)$$

$$= \frac{V}{S}\left(\frac{L/R}{S} - \frac{L/R}{S+R/L}\right)$$

$$= \frac{V}{\cancel{L}}\left(\frac{\cancel{L}}{R}\right)\left[\frac{1}{S} - \frac{1}{S+R/L}\right]$$

$$I(s) = \frac{V}{R}\left[\frac{1}{S} - \frac{1}{S+R/L}\right] \rightarrow ②$$

Eq ② Apply inverse laplace transform

$$i(t) = \frac{V}{R}\left[1 - e^{(-R/L)t}\right]$$

16-10-22

Transient response of R-C circuit:

Consider a circuit consisting of resistance in series with capacitance as shown in figure

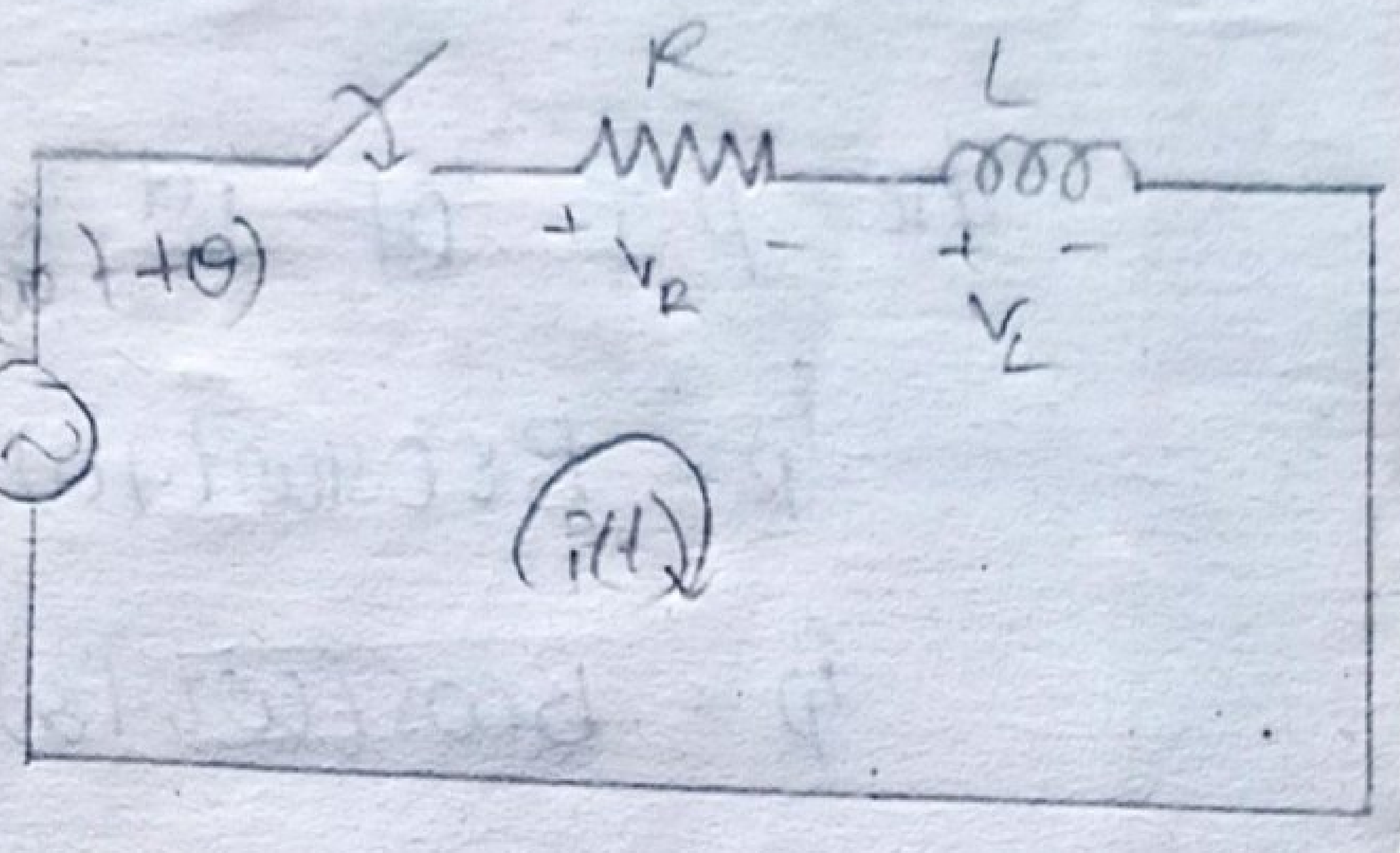
Apply KVL in given circuit

$$V = V_R + V_C$$

$$= iR + \frac{1}{C}$$

AC TRANSIENTS

1. **Sinusoidal responses of a RL circuit:**



Consider a series circuit consisting of resistance and inductance as shown in figure. The switch 'S' is closed at $t=0$. A sinusoidal voltage $V_m \sin\omega t$ is applied to the series RL circuit

Where V_m is the amplitude of the wave and θ is the phase angle

Apply KVL in above circuit

$$V_m \sin(\omega t+\theta) = V_R + V_L$$

$$V_m \sin(\omega t+\theta) = i(t) R + L\frac{di(t)}{dt}$$

$$L\left[\frac{di(t)}{dt} + \frac{R}{L} i(t)\right] = V_m \sin(\omega t+\theta)$$

$$\frac{di(t)}{dt} + \frac{R}{L} i(t) = \frac{V_m}{L}\sin(\omega t+\theta) \longrightarrow (1)$$

The above equation is linear differential equation of a first order

$\frac{d}{dt} = D$ as a operator

$$D i(t) + \frac{R}{L} i(t) = \frac{V_m}{L}(\sin(\omega t+\theta))$$

$$\frac{V_m}{L}\sin(\omega t+\theta) = i(t)\left[D+\frac{R}{L}\right]$$

The C.F is $\left[D+\frac{R}{L}\right] i(t) = 0$

$$\boxed{D = \frac{-R}{L}}$$

The Solution of C.F is = $Ae^{-\frac{R}{L}t}$.

$$CF = i_c = Ae^{-R/L\,t}.$$

The p.I of 1st order differential equation is

$$i_p = B\cos(\omega t+\theta) + C\sin(\omega t+\theta) \rightarrow (2)$$

$$i_p = B\cos(\omega t+\theta) + C\sin(\omega t+\theta)$$

$$\frac{di_p}{dt} = B(-\sin(\omega t+\theta)\omega) + C\cos(\omega t+\theta)\omega$$

$$= -B\sin(\omega t+\theta)\omega + C\omega\cos(\omega t+\theta) \rightarrow (3)$$

Sub (2) & (3) in eq (1)

$$\frac{V_m}{L}\sin(\omega t+\theta) = \frac{R}{L}\left[B\cos(\omega t+\theta) + C\sin(\omega t+\theta)\right] + \left[-B\omega\sin(\omega t + \theta) + C\omega\cos(\omega t+\theta)\right]$$

$$\frac{V_m}{L}\sin(\omega t+\theta) = \frac{R}{L}\left[B\cos(\omega t+\theta) + C\sin(\omega t+\theta)\right] + \left[-B\omega\sin(\omega t+\theta) + C\omega\cos(\omega t+\theta)\right]$$

$$\frac{V_m}{L}\sin(\omega t+\theta) = \sin(\omega t+\theta)\left[\frac{R}{L}C - B\omega\right) + \cos(\omega t+\theta)\left[\frac{R}{L}B + C\omega\right]$$

$$\frac{V_m}{L}\sin(\omega t+\theta) = \sin(\omega t+\theta)\left[\frac{R}{L}C - B\omega\right] + \cos(\omega t+\theta)\left[\frac{R}{L}B + C\omega\right]$$

Compare sin & cos terms

$$\frac{V_m}{L} = \frac{R}{L}C - B\omega \rightarrow (4)$$

$$0 = B\frac{R}{L} + C\omega$$

$$\frac{BR}{L} = -C\omega$$

$$B = -\frac{L}{R}C\omega \rightarrow (5)$$

B value in (4)

$$\frac{V_m}{L} = \frac{RC}{L} - B\omega$$

$$\frac{V_m}{L} = C\frac{R}{L} - \left(-\frac{L}{R}C\omega\right)\omega$$

$$\frac{V_m}{L} = C\frac{R}{L} + \frac{L}{R}C\omega^2$$

$$\frac{V_m}{L} = C\left[\frac{R}{L} + \frac{L}{R}\omega^2\right]$$

$$\frac{V_m}{\cancel{L}} = C\left[\frac{R^2+L^2\omega^2}{\cancel{L}R}\right]$$

$$V_m = C\left(\frac{R^2+L^2\omega^2}{R}\right)$$

$$C = \frac{V_m R}{R^2+L^2\omega^2}$$

from (5)

$$B = \frac{-L\omega}{\cancel{R}}\left[\frac{V_m \cancel{R}}{R^2+L^2\omega^2}\right]$$

$$B = -V_m\left(\frac{L\omega}{R^2+L^2\omega^2}\right)$$

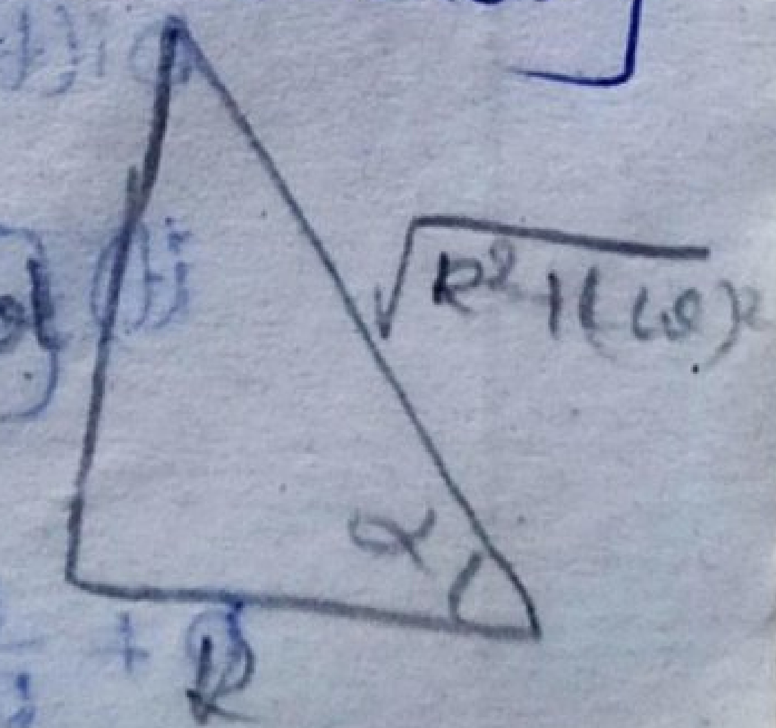
Sub B & C in eq (2)

$$i_p = \frac{-V_m(L\omega)}{(R^2+L^2\omega^2)}\cos(\omega t+\theta) + \frac{V_m R}{(R^2+L^2\omega^2)}\sin(\omega t+\theta)$$

$$\cancel{= \frac{V_m}{\sqrt{R^2+L^2\omega^2}}\left[L\omega \cos(\omega t+\theta)\right.}$$

$$i_p = \frac{V_m}{\sqrt{R^2+L^2\omega^2}}\left[\frac{-L\omega\cos(\omega t+\theta)}{\sqrt{R^2+(L\omega)^2}} + \frac{R}{\sqrt{R^2+(L\omega)^2}}\sin(\omega t+\theta)\right]$$

$$\sin\alpha = \frac{\omega L}{\sqrt{R^2+(L\omega)^2}}, \quad \cos\alpha = \frac{R}{\sqrt{R^2+(L\omega)^2}}$$



$$\Rightarrow i_p = \frac{V_m}{\sqrt{R^2+(L\omega)^2}}\left[-\sin\alpha\cos(\omega t+\theta) + \cos\alpha\sin(\omega t+\theta)\right]$$

$$= \frac{V_m}{\sqrt{R^2+(L\omega)^2}} \sin(\omega t+\theta-\alpha)$$

$$= \frac{V_m}{\sqrt{R^2+(L\omega)^2}} \sin(\omega t+\theta-\alpha)$$

* Sinusoidal response of R-C circuit:-

$$V_m\sin(\omega t+\theta) = V_R + V_C$$

$$V_m\sin(\omega t+\theta) = i R + \frac{1}{C}\int i(t)\,dt$$

(integrating on bot)

differentiating on both.

$$V_m\sin(\omega t+\theta) =$$

$$V_m\cos(\omega t+\theta)\omega = R\frac{di(t)}{dt} + \frac{1}{C}i(t)$$

$$V_m\omega\cos(\omega t+\theta) = R\frac{di(t)}{dt} + \frac{i(t)}{C}$$

$$\frac{V_m\omega}{R}\cos(\omega t+\theta) = \frac{di(t)}{dt} + \frac{1}{RC}i(t) \longrightarrow (1)$$

The above equation is in linear differential Equation of a first order

$$\frac{d}{dt} = D \text{ as operator}$$

$$Di(t) + \frac{1}{RC}i(t) = \frac{V_m\omega}{R}\cos(\omega t+\theta)$$

$$i(t)\left[D + \frac{1}{RC}\right] = \frac{V_m\omega}{R}\cos(\omega t+\theta)$$

$$D + \frac{1}{RC} = 0$$

$D = \frac{-1}{RC}$

The solution of C.F is $= \cancel{Ae^{-R}} \; Ae^{Dt}$

$= Ae^{\frac{-1}{RC}t}$

$C.F = i_c = Ae^{(\frac{-1}{RC})t}$

The P.I of first orde differential equation is

$i_p = B\cos(\omega t+\theta) + C\sin(\omega t+\theta) \longrightarrow ②$

$\frac{di_p}{dt} = B(-\sin(\omega t+\theta)\omega) + C\cos(\omega t+\theta)\omega$

$= -B\omega\sin(\omega t+\theta) + C\omega\cos(\omega t+\theta) \longrightarrow ③$

Sub ② & ③ in eq ①

$\frac{V_m\omega}{R}\cos(\omega t+\theta) = \frac{1}{RC}[B\cos(\omega t+\theta) + C\sin(\omega t+\theta)]$

$-B\omega\sin(\omega t+\theta) + C\omega(\cos(\omega t+\theta)$

$\frac{V_m\omega}{R}\cos(\omega t+\theta) = \sin(\omega t+\theta)\left[\frac{1}{RC}(C) - B\omega\right] + \cos(\omega t+\theta)\left(\frac{1}{RC}(B) + C\omega\right)$

$\frac{V_m\omega}{R}\cos(\omega t+\theta) = \sin(\omega t+\theta)\left[\frac{1}{RC}(C) - B\omega\right] + \cos(\omega t+\theta)\left[\frac{1}{RC}B + C\omega\right]$

compare Sin & cos

$\frac{V_m\omega}{R} = \frac{B}{RC} + C\omega \longrightarrow ④$

$0 = \frac{C}{RC} - B\omega \longrightarrow ⑤$

$B\omega = \frac{1}{RC}(C)$ constant $\longrightarrow$ Capacitor

$B = \frac{1}{RC}\frac{(C)}{\omega} \longrightarrow ⑤$

B value in eq (4)

$$\frac{V_m\omega}{R} = \frac{\left(\frac{C}{RC\omega}\right)}{RC} + C\omega$$

$$\frac{V_m\omega}{R} = \frac{C}{(RC)^2\omega} + C\omega$$

$$\frac{V_m\omega}{R} = C\left(\omega + \frac{1}{(RC)^2\omega}\right)$$

$$C = \frac{V_m\omega}{R\left(\omega + \frac{1}{(RC)^2\omega}\right)} = \frac{V_m \cancel{\omega}}{\cancel{\omega} R\left[1 + \frac{1}{(\omega RC)^2}\right]}$$

$$C = \frac{V_m\omega}{R\left(\omega + \frac{1}{(RC)^2\omega}\right)} = \frac{V_m\omega}{\frac{\cancel{R}}{R^2}\left[R^2 + \frac{1}{(\omega C)^2}\right]}$$

$$\therefore B = \frac{1}{RC\omega}(C)$$

$$B = \frac{1}{RC\omega} \cdot \frac{V_m\omega}{R\left(\omega + \frac{1}{(RC)^2\omega}\right)}$$

$$\boxed{C = \frac{V_m R}{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$

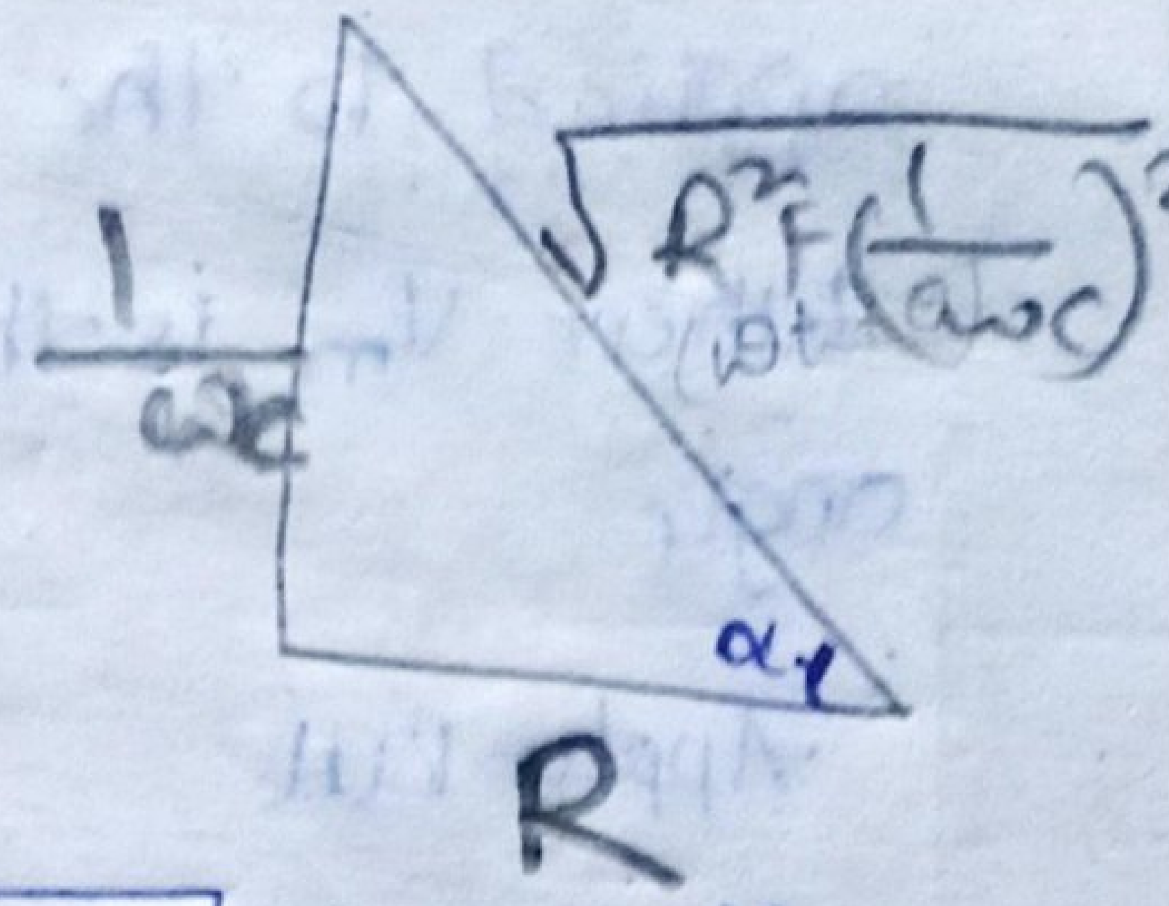
$$B = \frac{V_m}{R^2C\left(\omega + \frac{1}{(RC)^2\omega}\right)}$$

Sub B & C value in eq (2)

$$i_p = \frac{V_m}{R^2C\left(\omega + \frac{1}{(RC)^2\omega}\right)}\cos(\omega t + \theta) + \frac{V_m\omega}{R\left(\omega + \frac{1}{(RC)^2\omega}\right)}\sin(\omega t + \theta)$$

$$i_p = \frac{V_m}{R\left(\omega + \frac{1}{(RC)^2\omega}\right)}\left[\frac{\cos(\omega t + \theta)}{RC} + \omega\sin(\omega t + \theta)\right]$$

$$B = \frac{V_m \cancel{R}}{\omega \cancel{R} C\left(R^2 + \left(\frac{1}{\omega C}\right)^2\right)}$$



$$\boxed{B = \frac{V_m}{\omega C\left(R^2 + \left(\frac{1}{\omega C}\right)^2\right)}}$$

$\sqrt{2}\times\sqrt{2} = 2$

$$i_p = \frac{V_m}{\omega C\left(R^2+\left(\frac{1}{\omega C}\right)^2\right)}\cos(\omega t+\theta) + \frac{V_m R}{R^2+\left(\frac{1}{\omega C}\right)^2}\sin(\omega t+\theta)$$

$$i_p = \frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}\left[\frac{1}{\omega C}\frac{\cos(\omega t+\theta)}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}} + \frac{R\sin(\omega t+\theta)}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}\right]$$

from the diagram:

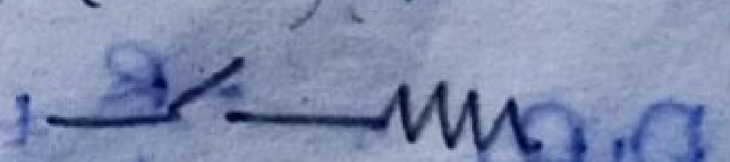
$$\sin\alpha = \frac{\frac{1}{\omega C}}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}} \qquad \cos\alpha = \frac{R}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}$$

$$i_p = \frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}\left[\sin\alpha\cos(\omega t+\theta) + \cos\alpha\sin(\omega t+\theta)\right]$$

$$= \frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}\left[\sin(\omega t+\theta+\alpha)\right]$$

Sinusoidal response of R-L-C circuit

Consider a series circuit consisting of resistance, inductance, capacitance as shown in fig. The switch S is closed at $t=0$. A ~~simult~~ (-aneo) sinusoidal voltage ~~V~~ $V_m\sin\omega t$



applied to the R-L-C circuit

where V_m is the amplitude & θ is the phase angle

Apply KVL

$V_m\sin(\omega t+\theta) = V_R+V_L+V_C$

$V_m\sin(\omega t+\theta) = i(t)R + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt$

differentating on both sides

$V_m\cos(\omega t+\theta) = R\frac{di(t)}{dt} + L\frac{d^2i(t)}{dt^2} + \frac{1}{C}i(t)$

$\frac{V_m\cos(\omega t+\theta)}{L} = \frac{R}{L}\frac{di(t)}{dt} + \frac{d^2i(t)}{dt^2} + \frac{1}{LC}i(t)$

$\frac{d}{dt} = D$ as operator

$D^2 i(t) + \frac{R}{L}D\, i(t) + \frac{1}{LC}i(t) = \frac{V_m}{L}\cos(\omega t+\theta) \rightarrow$ (I)

$i(t)\left[D^2+\frac{R}{L}D+\frac{1}{LC}\right] = \frac{V_m}{L}\cos(\omega t+\theta)$

$D^2+\frac{R}{L}D+\frac{1}{LC} = 0$

$D_1, D_2 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{+R}{L}\right)^2 - 4(1)\left(\frac{1}{LC}\right)}}{2}$

$D_1, D_2 = \frac{-\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - \frac{4}{LC}}}{2}$

$D_1, D_2 = \frac{-\frac{R}{L} \pm \frac{R}{L}\sqrt{1-\frac{4L}{R^2C}}}{2}$ $\qquad \alpha = \frac{-R}{2L}$

$= \frac{-R}{2L} \pm \sqrt{\frac{R^2}{2^2L^2} - \frac{4}{4LC}}$ $\qquad \beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

$= \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

Solution of C.F is $i_c = k_1 e^{D_1 t} + k_2 e^{D_2 t}$

k_1 & k_2 are the constants

D_1, D_2 are the roots

case i, when

$$\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$$

At this time β is positive real quantity

Hence the roots are $D_1 D_2$ are real & unequal

$$D_1 = \alpha + \beta \quad D_2 = \alpha - \beta$$

The current solution is

$$i = k_1 e^{D_1 t} + k_2 e^{D_2 t}$$

$$i = k_1 e^{(\alpha+\beta)t} + k_2 e^{(\alpha-\beta)t}$$

$$i = k_1 e^{\alpha t} e^{\beta t} + k_2 e^{\alpha t} e^{-\beta t}$$

$$i = e^{\alpha t}\left[k_1 e^{\beta t} + k_2 e^{-\beta t}\right]$$

The above equation is the current soluti

ii, when $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$

at this time β is imaginary quantity & the roots $D_1 D_2$ are complex conjugate

$$D_1 = \alpha + j\beta \quad D_2 = \alpha - j\beta$$

$$i = k_1 e^{D_1 t} + k_2 e^{D_2 t}$$

$$i = k_1 e^{(\alpha+j\beta)t} + k_2 e^{(\alpha-j\beta)t}$$

$$i = k_1 e^{\alpha t} e^{j\beta t} + k_2 (e^{\alpha t} \cdot e^{-j\beta t})$$

$$i = e^{\alpha t}\left[k_1 e^{j\beta t} + k_2 e^{-j\beta t}\right]$$

$$i = e^{\alpha t}\left[k_1 e^{j\beta t} + k_2 e^{-j\beta t}\right]$$

current damped
The Solution is oscillatory & underguamnt

Case iii,

$$\frac{R}{LQ} = \frac{1}{LC}$$

$D_1 = \alpha + \beta \quad D_2 = \alpha - \beta$

at this time β is zero Hence D_1, D_2 are the real & equal

$D_1 = \alpha + 0 \qquad D_2 = \alpha + 0$

$D_1 = \alpha \qquad D_2 = \alpha$

$$i = k_1 e^{\alpha t} + k_2 e^{\alpha t}$$

$$i = e^{\alpha t}[k_1 + k_2]$$

The Solution is critically damped

The p.I Solution is

$$i_p = B\cos(\omega t + \theta) + C\sin(\omega t + \theta) \longrightarrow (1)$$

$$\frac{di_p}{dt} = B(-\sin(\omega t + \theta))\omega + C\cos(\omega t + \theta)\omega$$

$$\frac{di_p}{dt} = -B\sin(\omega t + \theta)\omega + C\cos(\omega t + \theta)\omega \longrightarrow (2)$$

$$\frac{d^2 i_p}{dt^2} = -B\omega\cos(\omega t + \theta)\omega + C\omega[-\sin(\omega t + \theta)]\omega$$

$$\frac{d^2 i_p}{dt^2} = -B\omega^2\cos(\omega t + \theta) - C\omega^2\sin(\omega t + \theta)$$

from the eq (I)

$$-B\omega^2\cos(\omega t + \theta) - C\omega^2\sin(\omega t + \theta) + \frac{R}{L}\left[-B\omega\sin(\omega t + \theta) + C\omega\cos(\omega t + \theta)\right]$$
$$+ \frac{1}{LC}\left[B\cos(\omega t + \theta) + C\sin(\omega t + \theta)\right] = \frac{V_m}{L}\cos(\omega t + \theta)$$

Compare cos terms & sin terms

$$\frac{V_m}{L} = -B\omega^2 + \frac{R\omega}{L}C + \frac{1}{LC}B \longrightarrow ③$$

$$\frac{V_m}{L} = B\left[\frac{1}{LC} - \omega^2\right] + C\frac{R\omega}{L} — ④ \qquad D = \frac{d}{dt}$$

$$B\left[\frac{1}{LC} - \omega^2\right] = \frac{V_m}{L} - \frac{CR\omega}{L} \qquad D^2 i(t)$$

$$\frac{d^2 i(t)}{dt^2}$$

$$B = \frac{\left(\frac{V_m - CR\omega}{L}\right)}{\left(\frac{1}{LC} - \omega^2\right)}$$

$$B = \frac{\cancel{L}(V_m - CR\omega)}{\cancel{L}\left(\frac{1}{C} - L\omega^2\right)} \Rightarrow B = \frac{\omega\left[\frac{V_m}{\omega} - CR\right]}{\omega^2\left[\frac{1}{C\omega^2} - L\right]}$$

$$B = \frac{\left[\frac{V_m}{\omega} - CR\right]}{\omega\left[\frac{1}{C\omega^2} - L\right]}$$

Compare sin terms

$$-C\omega^2 - \frac{R\omega}{L}B + \frac{1}{LC}C = 0 \longrightarrow ⑤$$

$$C\left[\frac{1}{LC} - \omega^2\right] = \frac{R\omega}{L}B$$

$$B = C\left[\frac{1}{LC} - \omega^2\right]\left[\frac{L}{R\omega}\right]$$

from eq ④

$$-C\omega^2 - \frac{R\omega}{L}\left[\frac{L}{R\omega}\right]\left[C\left[\frac{1}{LC} - \omega^2\right]\right] + \frac{1}{LC}C = 0$$

$$C\left[-\omega - \frac{1}{LC} + \omega^2\right] + \frac{1}{LC}C = 0$$

$$-\frac{1}{LC}C + \frac{1}{LC}C = 0$$

from eq (3) Sub B

$$\frac{V_m}{L} = C\left[\frac{1}{LC}-\omega^2\right]\left[\frac{1}{R\omega}\right]\left[\frac{1}{LC}-\omega^2\right] + C\frac{R\omega}{L}$$

$$\frac{V_m}{L} = C\left[\left(\frac{1}{LC}-\omega^2\right)^2\left(\frac{L}{R\omega}\right)\right] + C\frac{R\omega}{L}$$

$$\frac{V_m}{L} = C\left(\left[\frac{1}{LC}-\omega^2\right]^2\left[\frac{L}{R\omega}\right] + \frac{R\omega}{L}\right)$$

$$\boxed{C = \frac{V_m}{L}\left[\frac{1}{\left(\frac{1}{LC}-\omega^2\right)^2\left(\frac{L}{R\omega}\right)+\left(\frac{R\omega}{L}\right)}\right]}$$

$$B = \frac{V_m}{L}\left[\frac{1}{\left(\frac{1}{LC}-\omega^2\right)^2\left(\frac{L}{R\omega}\right)+\left(\frac{R\omega}{L}\right)}\right]\left[\frac{1}{LC}-\omega^2\right]\left(\frac{L}{R\omega}\right)$$

$$\boxed{B = \frac{V_m\left(\frac{1}{LC}-\omega^2\right)}{\left[\left(\frac{1}{LC}-\omega^2\right)^2\left(\frac{L}{R\omega}\right)+\frac{R\omega}{L}\right]}}$$

Badhaga untadi

Dopam kanna

$$i_p = \left[\frac{V_m\left(\frac{1}{LC}-\omega^2\right)}{\left(\frac{1}{LC}-\omega^2\right)^2\frac{L}{R\omega}+\frac{R\omega}{L}}\right]\cos(\omega t+\theta) + \frac{V_m}{L}\left[\frac{1}{\left(\frac{1}{LC}-\omega^2\right)^2\frac{L}{\omega R}+\frac{R\omega}{L}}\right]\times \sin(\omega t+\theta)$$

$$i_p = \frac{V_m\left(\frac{1}{LC}-\omega^2\right)}{\frac{L}{R\omega}\left[\left(\frac{1}{LC}-\omega^2\right)^2+\frac{R^2\omega^2}{L^2}\right]}\cos(\omega t+\theta) + \frac{V_m}{L}\frac{1}{\left[\left(\frac{1}{LC}-\omega^2\right)^2+\left(\frac{R\omega}{L}\right)^2\right]\frac{L}{R\omega}}$$

$= V_m$

2-11-22

UNIT-IV Network Parameters

Two port network.

Generally any network may be represented by a rectangular box

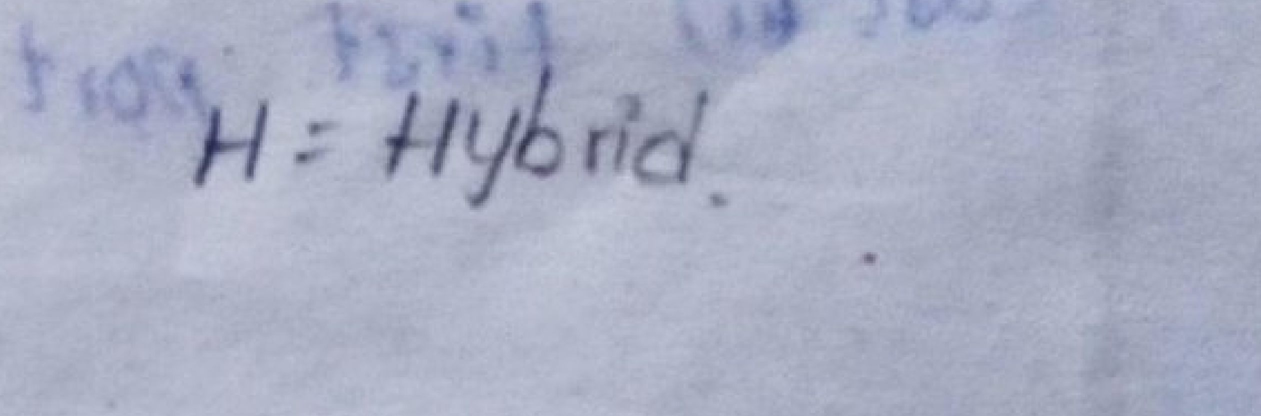
-A network may be represented either source or load or different purpose

port:- -A pair of terminals at which a signal may enter or leave a network is called a port.

Two port network:-

-A Two port network or two terminals pair of network in which 4 terminals

The port 1-1' voltage and current at the input terminals V_1 & I_1 the 2-2' is the another port is out put terminals



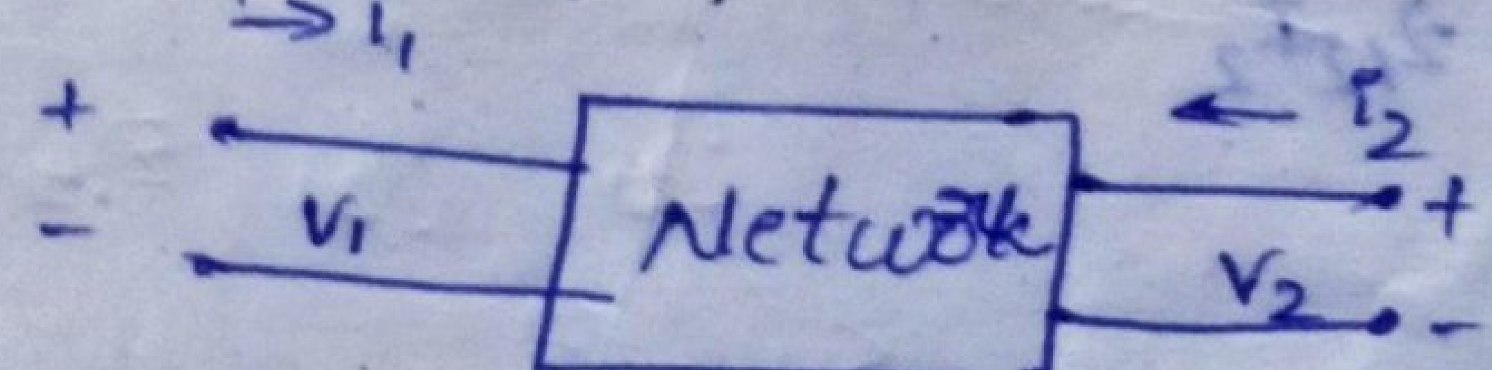
Rectangular box

→ Types of 2-port n/w paramters

1. Z-parameters
2. Y-parameter
3. ABCD-parmater
4. H-parameters
5. H^{-1} - parameter (Inverse H)

H = Hybrid.

Z-parameters / open circuit parameters



$V = IZ$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

[V is dependent
I is independent]

$V_1 = Z_{11} I_1 + Z_{12} I_2 \longrightarrow (1)$

$V_2 = Z_{21} I_1 + Z_{22} I_2 \longrightarrow (2)$

Case i: 2nd port is open $I_2 = 0$

$(1) \Rightarrow V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_1 = Z_{11} I_1 + Z_{12}(0)$

$V_1 = Z_{11} I_1 + 0$

$V_1 = Z_{11} I_1$ ∴ Z_{11} drawing point impedance at port 1

$Z_{11} = \frac{V_1}{I_1}$

$(2) \Rightarrow V_2 = Z_{21} I_1 + Z_{22} I_2$

$V_2 = Z_{21} I_1 + Z_{22}(0)$

$V_2 = Z_{21} I_1 + 0$

$Z_{21} = \frac{V_2}{I_1}$

Case (ii) first port is open $I_1 = 0$

$(1) \Rightarrow V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_1 = Z_{11}(0) + Z_{12} I_2$

$V_1 = Z_{12} I_2$

$\therefore Z_{12} = \frac{V_1}{I_2}$

Reciprocity:-

$y_{12} = y_{21}$ $Z_{12} = Z_{21}$

Symetry

$y_{11} = y_{22}$ $Z_{11} = Z_{22}$

$(2) \Rightarrow V_2 = Z_{21} I_1 + Z_{22} I_2$

$V_2 = Z_{21}(0) + Z_{22} I_2$

$V_2 = z_{22} I_2$

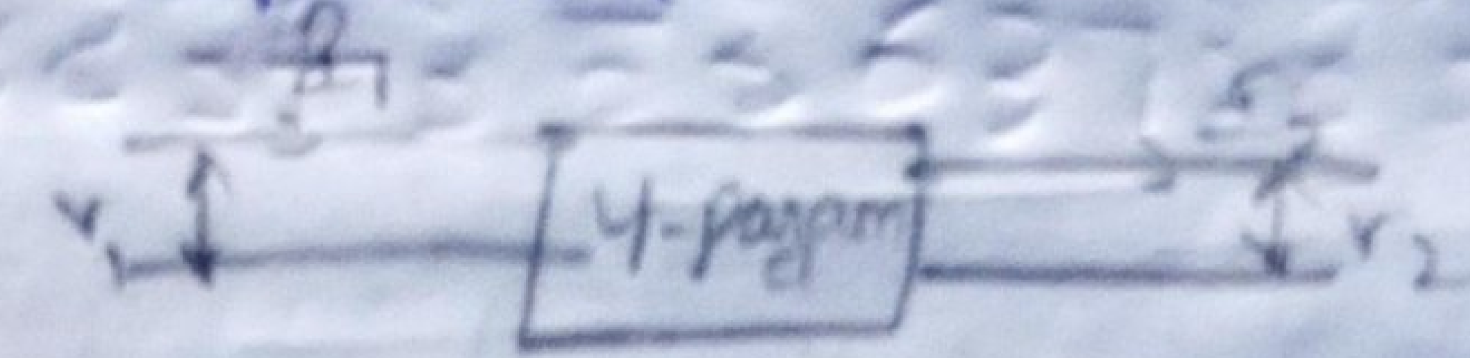
$z_{22} = \frac{V_2}{I_2}$ [∵ z_{22} driving point impedence at port 2]

∴ z_{21} Open circuit forward transform impedence

2 Y parameters (or) short circuit parameters

I = dependent $V = I$

V = independent



$V = IZ$ ∵ $Z = \frac{1}{Y}$

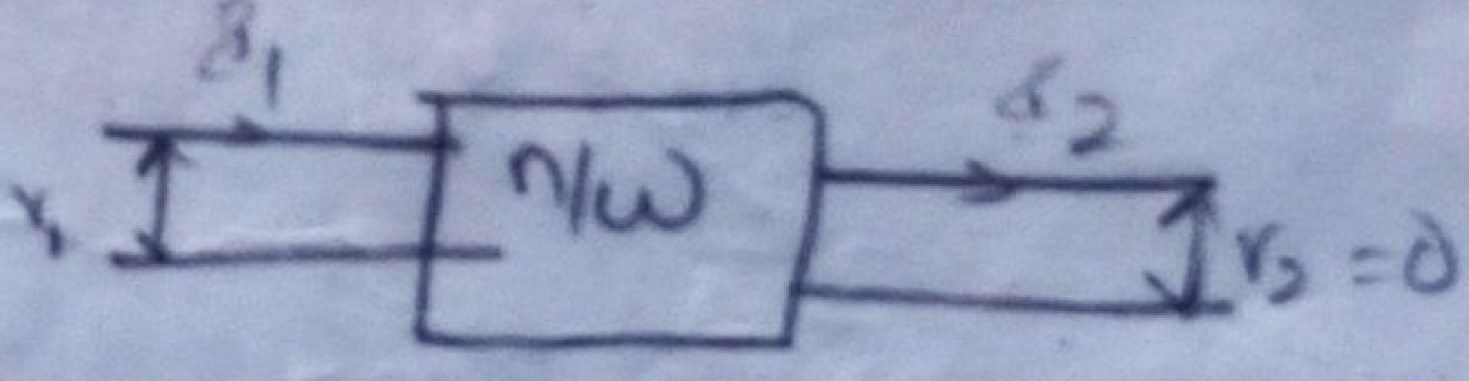
$V = I \frac{1}{Y}$ ⇒ $I = VY$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$I_1 = y_{11} V_1 + y_{12} V_2$

$I_2 = y_{21} V_1 + y_{22} V_2$

case (i) IInd port is short circuit, i.e $V_2 = 0$



$I_1 = y_{11} V_1 + y_{12}(0)$

$I_1 = y_{11} V_1 + 0$

$y_{11} = \frac{I_1}{V_1}$

∵ y_{11} driving point ~~adm~~ admitance at port 1

$② \Rightarrow I_2 = y_{21}V_1 + y_{22}(V_2)$

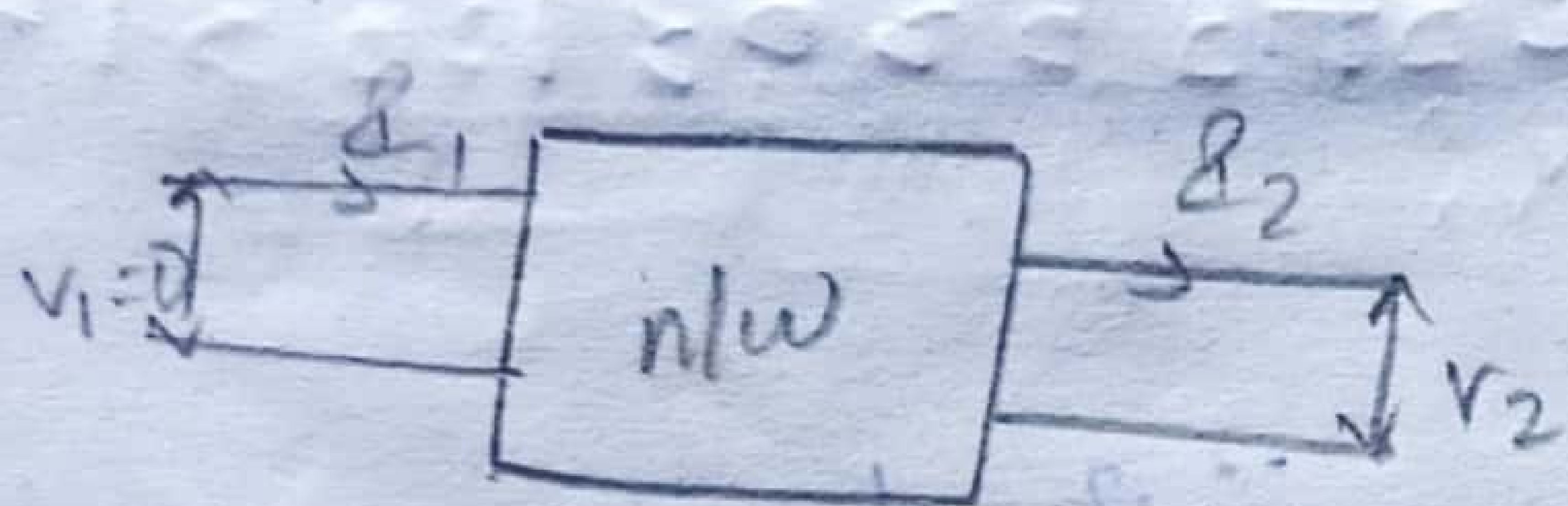
$I_2 = y_{21}V_1 + y_{22}(0)$

$I_2 = y_{21}V_1 + 0$

$y_{21} = \frac{I_2}{V_1}$

$\therefore y_{21}$ Short circuit forword transform admittance

case (ii), 1st part is short circuit. i.e $V_1 = 0$.



$I_1 = y_{11}(0) + y_{12}V_2$

$I_1 = y_{12}V_2$

$y_{12} = \frac{I_1}{V_2}$

$\therefore y_{12}$ short circuit reverse transform admittance

$② \Rightarrow I_2 = y_{21}(0) + y_{22}V_2$

$I_2 = y_{22}V_2$

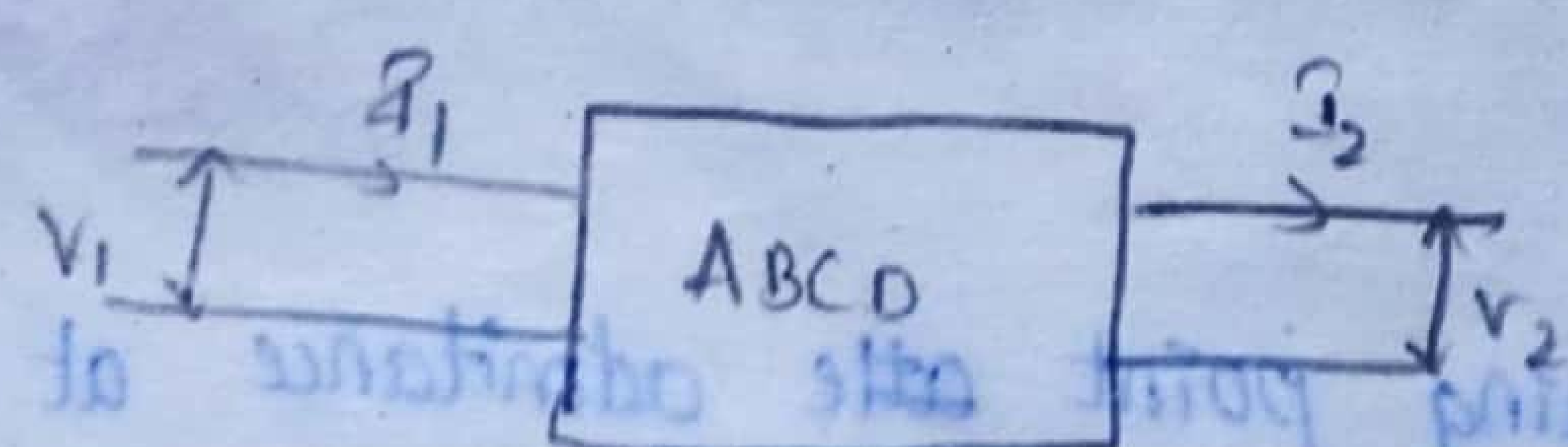
$y_{12} = y_{21}$

$y_{22} = \frac{I_2}{V_2}$

$y_{11} = y_{22}$

y_{22} driving point admittance at port 2

③ ABCD parameter / transmachine line parameter

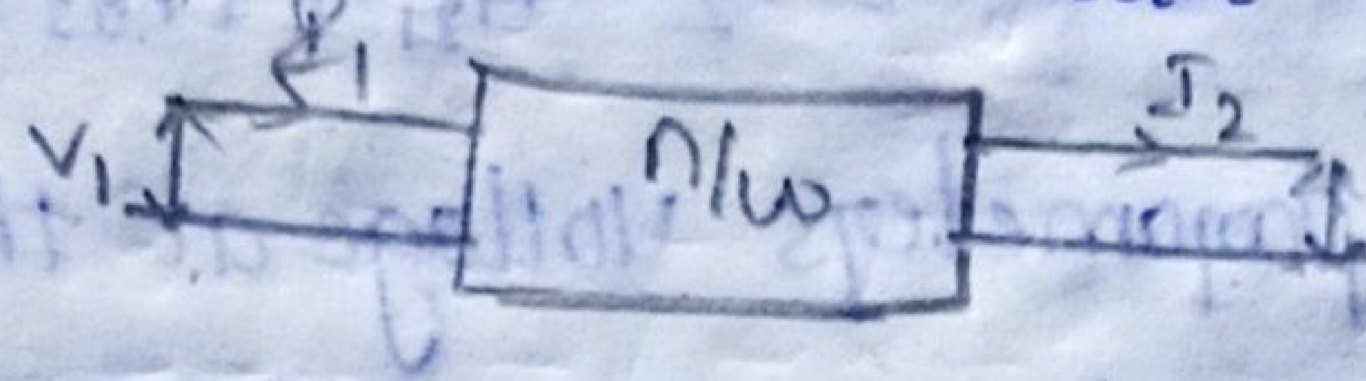


$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$V_1 = AV_2 + B(-I_2) \longrightarrow (1)$$

$$I_1 = CV_2 + D(-I_2) \longrightarrow (2)$$

case (i) port is short circuit i.e. $V_2 = 0$



$(1) \Rightarrow V_1 = AV_2 + B(-I_2)$

$$V_1 = 0 + B(-I_2)$$

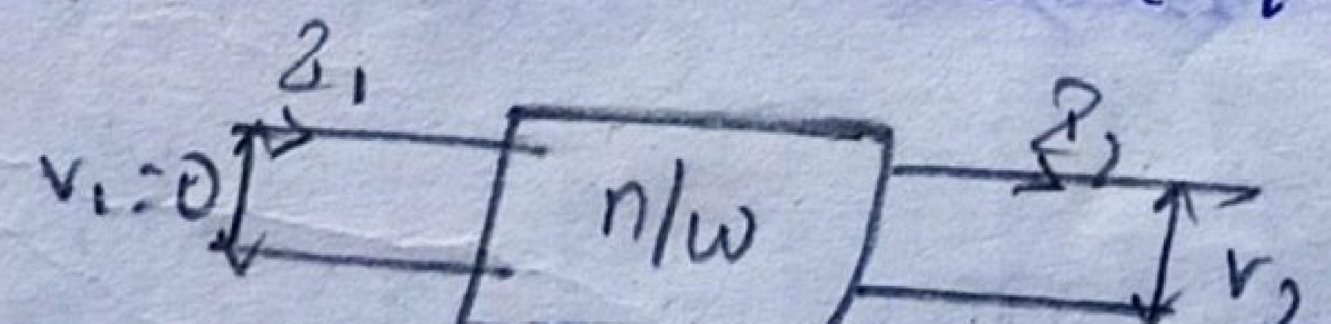
$$B = \frac{V_1}{-I_2}$$

$(2) \Rightarrow I_1 = CV_2 + D(-I_2)$

$$I_1 = 0 + D(-I_2)$$

$$D = \frac{I_1}{-I_2}$$

case (ii) port is short circuit i.e. $I_2 = 0$



$(1) \rightarrow V_1 = AV_2 + B(-I_2)$

$$V_1 = AV_2 + B(0)$$

$$V_1 = AV_2$$

$$A = \frac{V_1}{V_2}$$

$AD - BC = 1$

$A = D$

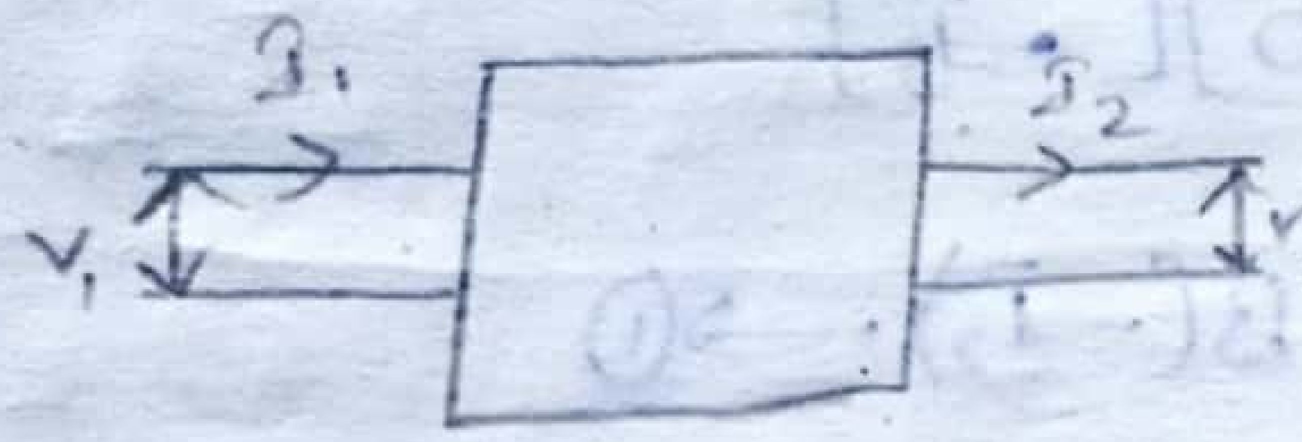
$(2) \Rightarrow I_1 = CV_2 + D(-I_2)$

$$I_1 = CV_2 + D(0)$$

$$I_1 = CV_2$$

$$C = I_1 / V_2$$

H- parameters:



$\cancel{V_1 = h_{11}V_2 + h_{12}I_1}$ $\quad V_1 = h_{11}I_1 + h_{12}V_2$

$\cancel{I_2 = h_{21}V_2 + h_{22}I_1}$ $\quad I_2 = h_{21}I_1 + h_{22}V_2$

In hybrid parameters voltage at the transmitting end & current at the receiving end are expressed in in-terms of voltage & current at a receiving end and transmitting end respectively

Case (i) 2nd short ckt i.e $V_2 = 0$

$① \rightarrow V_1 = h_{11}I_1 + h_{12}(0)$ $\qquad ② \Rightarrow$

$V_1 = h_{11}I_1$

$h_{11} = \frac{V_1}{I_1}$

$② \Rightarrow I_2 = h_{21}I_1 + h_{22}(0)$

$I_2 = h_{21}I_1$

$h_{21} = \frac{I_2}{I_1}$

Case (ii) 1st port is open $I_1 = 0$

$① \Rightarrow V_1 = h_{11}I_1 + h_{12}V_2$

$V_1 = h_{11}(0) + h_{12}V_2$

$V_1 = h_{12}V_2$

$h_{12} = \frac{V_1}{V_2}$

$② \Rightarrow I_2 = h_{21}(0) + h_{22}V_2$

$I_2 = h_{22}V_2$

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$$h_{22} = \frac{I_2}{V_2}$$

Inverse transmission line / Inverse ABCD parameters

$$V_2 = A'V_1 + B'(-I_1) \longrightarrow (1)$$

$$I_2 = C'V_1 + D'(-I_1) \longrightarrow (2)$$

case i, port is short $V_1 = 0$

$$V_2 = A'(0) - B'I_1$$

$$B' = \frac{-V_2}{I_1}$$

$(2) \Rightarrow I_2 = C'(0) - D'I_1$

$$D = \frac{I_2}{I_1}$$

case (ii) $I_1 = 0$

$(1) \Rightarrow V_2 = A'V_1 + B'(0)$

$$V_2 = A'V_1$$

$$A' = \frac{V_2}{V_1}$$

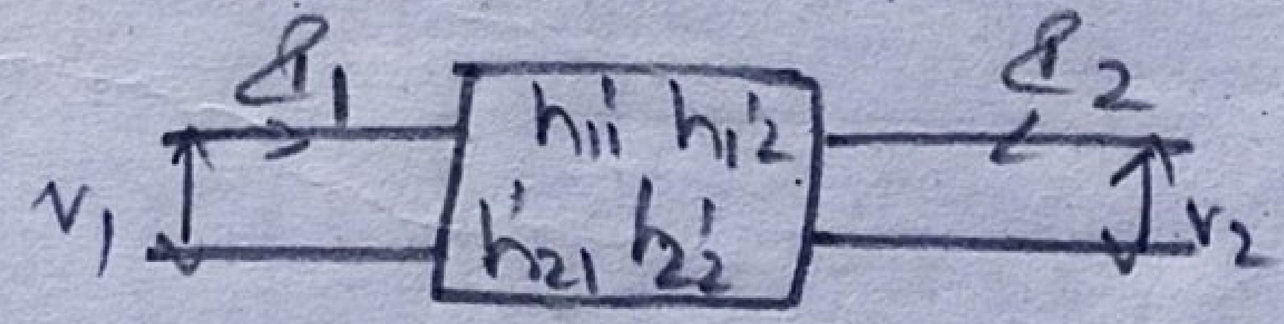
$(2) \Rightarrow I_2 = C'V_1 - D'(0)$

$$C' = \frac{I_2}{V_1}$$

$$A'D' - B'C' = 1$$

$$A' = D'$$

Inverse H parameters



$$I_1 = h_{11}'V_1 + h_{12}'I_2$$

$$V_2 = h_{21}'V_1 + h_{22}'I_2$$

case (i) 1st port is short $V_1 = 0$

$(1) \Rightarrow I_1 = h_{11}'(0) + h_{12}'I_2$

$$h_{12}' = \frac{I_1}{I_2}$$

② ⇒ $V_2 = h'_{21}(0) + h'_{22} I_2$

$V_2 = h'_{22} I_2$

$h'_{22} = \frac{V_2}{I_2}$

case (ii) 2nd port is open ckt i.e. $I_2 = 0$

① ⇒ $I_1 = h'_{11} V_1 + h'_{12} I_2$

$V_2 = h'_{21} V_1 + h'_{22} I_2$

① ⇒ $I_1 = h'_{11}(V_1) + h'_{12}(0)$

$h'_{11} = \frac{I_1}{V_1}$

② ⇒ $V_2 = h'_{21}(V_1) + h'_{22}(0)$

$h'_{21} = \frac{V_2}{V_1}$

$h = g$

$h_{21} = -h_{12}$

$h_{11} h_{22} = -h_{12} h_{21}$

Inter relationship between different parameters:-

1. Y-parameters in terms of Z-parameters:-

$V = IZ \Rightarrow V = I\left[\frac{1}{Y}\right] \quad \because Z = \frac{1}{Y}$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2.$

$$\therefore I_1 = \frac{\left|\begin{bmatrix} V_1 & Z_{12} \\ V_2 & Z_{22} \end{bmatrix}\right|}{\Delta Z}$$

$$I_1 = \frac{V_1 Z_{22} - V_2 Z_{12}}{\Delta Z}$$

$$I_1 = V_1\left[\frac{Z_{22}}{\Delta Z}\right] + V_2\left[\frac{-Z_{12}}{\Delta}\right] \Rightarrow V_1\left[\frac{-Z_{22}}{\Delta Z}\right] \pm V_2\left[\frac{-Z_{12}}{\Delta Z}\right]$$

$I_1 = V_1 y_{11} + V_2 y_{12}$

$\therefore y_{11} = \frac{z_{22}}{\Delta z} \quad y_{12} = \frac{-z_{12}}{\Delta z}$

from $I_2 = \frac{\begin{vmatrix} z_{11} & V_1 \\ z_{21} & V_2 \end{vmatrix}}{\Delta z}$

$$I_2 = \frac{z_{11} V_2 - z_{21} V_1}{\Delta z}$$

$$I_2 = V_2 \left[\frac{z_{11}}{\Delta z}\right] + V_1 \left[\frac{-z_{21}}{\Delta z}\right]$$

$$I_2 = V_1 \left[\frac{-z_{21}}{\Delta z}\right] + V_2 \left[\frac{z_{11}}{\Delta z}\right]$$

$\therefore I_2 = V_1 y_{21} + V_2 y_{22}$

$\therefore y_{21} = \frac{-z_{21}}{\Delta z} \quad y_{22} = \frac{z_{11}}{\Delta z}$

$y = \frac{1}{z}$

$z^{-1} = y$

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}^{-1}$$

$$= \frac{1}{\Delta z} \begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}$$

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} \frac{z_{22}}{\Delta z} & -\frac{z_{12}}{\Delta z} \\ \frac{-z_{21}}{\Delta z} & \frac{z_{11}}{\Delta z} \end{bmatrix} \quad [\Delta z = z_{11} z_{22} - z_{12} z_{21}],$$

$y_{11} = \frac{z_{22}}{\Delta z} \quad y_{12} = \frac{-z_{12}}{\Delta z}$

$y_{21} = \frac{-z_{21}}{\Delta z} \quad y_{22} = \frac{z_{11}}{\Delta z}.$

Z parameters in terms of Y-parameters

$V = IZ \Rightarrow V = I\frac{1}{Y}$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$I_1 = y_{11}V_1 + y_{12}V_2$

$I_2 = y_{21}V_2 + y_{22}V_2$

$$V_1 = \frac{\begin{vmatrix} I_1 & y_{12} \\ I_2 & y_{22} \end{vmatrix}}{\Delta y}$$

$$V_1 = \frac{I_1 y_{22} - I_2 y_{12}}{\Delta y}$$

$$V_1 = I_1\left[\frac{y_{22}}{\Delta y}\right] + I_2\left[\frac{-y_{12}}{\Delta y}\right]$$

$V_1 = I_1 z_{11} + I_2 z_{12}$

$\therefore z_{11} = \frac{y_{22}}{\Delta y} \quad z_{12} = \frac{-y_{12}}{\Delta y}$

$$V_2 = \frac{\begin{vmatrix} y_{11} & I_1 \\ y_{21} & I_2 \end{vmatrix}}{\Delta y}$$

$$V_2 = \frac{y_{11} I_2 - I_1 y_{21}}{\Delta y}$$

$$V_2 = I_1\left[\frac{-y_{21}}{\Delta y}\right] + I_2\left[\frac{y_{11}}{\Delta y}\right]$$

$V_2 = I_1 z_{21} + I_2 z_{22}$

$\therefore z_{21} = \frac{-y_{21}}{\Delta y} \quad z_{22} = \frac{y_{11}}{\Delta y}$

Z parameters in terms of ABCD - parameters.

ABCD parameters

$V_1 = AV_2 + B(-I_2) \rightarrow ①$

$I_1 = CV_2 + D(-I_2) \rightarrow ②$

Converts Z - parameter.

$V_1 = Z_{11}I_1 + Z_{12}I_2 \rightarrow ③$

$V_2 = Z_{21}I_1 + Z_{22}I_2 \rightarrow ④$

$② \Rightarrow CV_2 = I_1 + DI_2$

$V_2 = I_1\left(\frac{1}{C}\right) + I_2 \frac{D}{C} \rightarrow ⑤$

Compare above eq ⑤ into eq ④

$Z_{21} = \frac{1}{C} \quad Z_{22} = \frac{D}{C}$

$① \Rightarrow$ sub eq ⑤ in eq ①

$V_1 = A\left[I_1\left(\frac{1}{C}\right) + I_2\left(\frac{D}{C}\right)\right] + B(-I_2)$

$V_1 = AI_1\left(\frac{1}{C}\right) + I_2\left(\frac{AD}{C} - B\right)$

$V_1 = I_1\left(\frac{A}{C}\right) + I_2\left(\frac{AD}{C} - B\right)$

$Z_{11} = \frac{A}{C} \quad Z_{12} = \frac{AD}{C} - B$

Z parameters in terms of H - parameters

$V_1 = h_{11}I_1 + h_{12}V_2 \rightarrow ①$

$I_2 = h_{21}I_1 + h_{22}V_2 \rightarrow ②$

Converts Z - parameter

$V_1 = Z_{11}I_1 + Z_{12}I_2 \rightarrow ③$

$V_2 = Z_{21}I_1 + Z_{22}I_2 \rightarrow ④$

$② \Rightarrow h_{22}V_2 = I_2 - h_{21}I_1$

$h_{22}V_2 = I_1[-h_{21}] + I_2$

$$V_2 = I_1\left[\frac{-h_{21}}{h_{22}}\right] + I_2\left[\frac{1}{h_{22}}\right] \rightarrow 5$$

$$\therefore z_{21} = \frac{-h_{21}}{h_{22}} \qquad z_{22} = \frac{1}{h_{22}}$$

1 ⇒ sub eq 5 in eq 1

$$V_1 = h_{11} I_1 + h_{12}\left[I_1\left(\frac{-h_{21}}{h_{22}}\right) + I_2\left[\frac{1}{h_{22}}\right]\right]$$

$$V_1 = \left(h_{11} - \frac{h_{12}h_{21}}{h_{22}}\right) I_1 + \left(\frac{h_{12}}{h_{22}}\right) I_2$$

$$z_{11} = h_{11} - \frac{h_{12}h_{21}}{h_{22}} \qquad z_{22} = \frac{h_{12}}{h_{22}}$$

y-parameters in terms of ABCD-parameters:

ABCD parameters.

$$V_1 = AV_2 + B(-I_2) \longrightarrow 1$$

$$I_1 = CV_2 + D(-I_2) \longrightarrow 2$$

converts of y-parameters

$$I_1 = y_{11}V_1 + y_{12}V_2 \longrightarrow 3$$

$$I_2 = y_{21}V_1 + y_{22}V_2 \longrightarrow 4$$

1 → $V_1 = AV_2 + B(-I_2)$

$$V_1 - AV_2 = -BI_2$$

$$BI_2 = AV_2 - V_1$$

$$I_2 = V_1\left(\frac{-1}{B}\right) + V_2\left(\frac{A}{B}\right) \rightarrow 5 \text{ compare with eq } 4$$

$$I_2 = V_1 y_{21} + y_{22}V_2$$

$$y_{21} = \frac{-1}{B} \qquad y_{22} = \frac{A}{B}$$

eq 5 in eq 1

28-11-22

UNIT-V

FILTERS

Definition:-

An electric filter is a circuit which can be design to Modify, reshape or reject all the desired undesigned frequency of an Electrical signal and pass only the disered signals

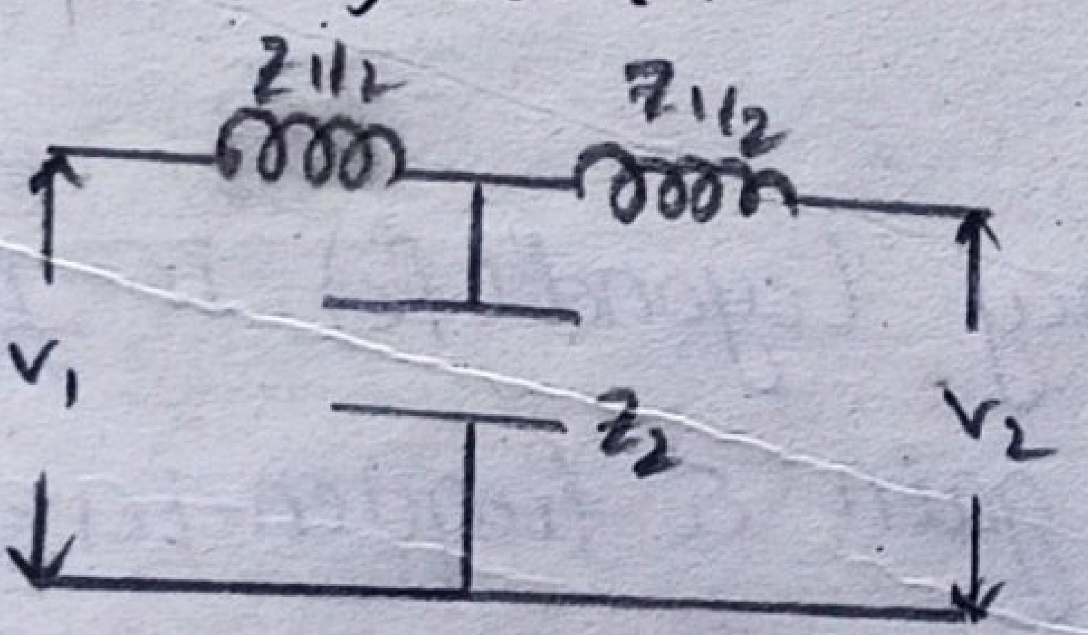
(or)

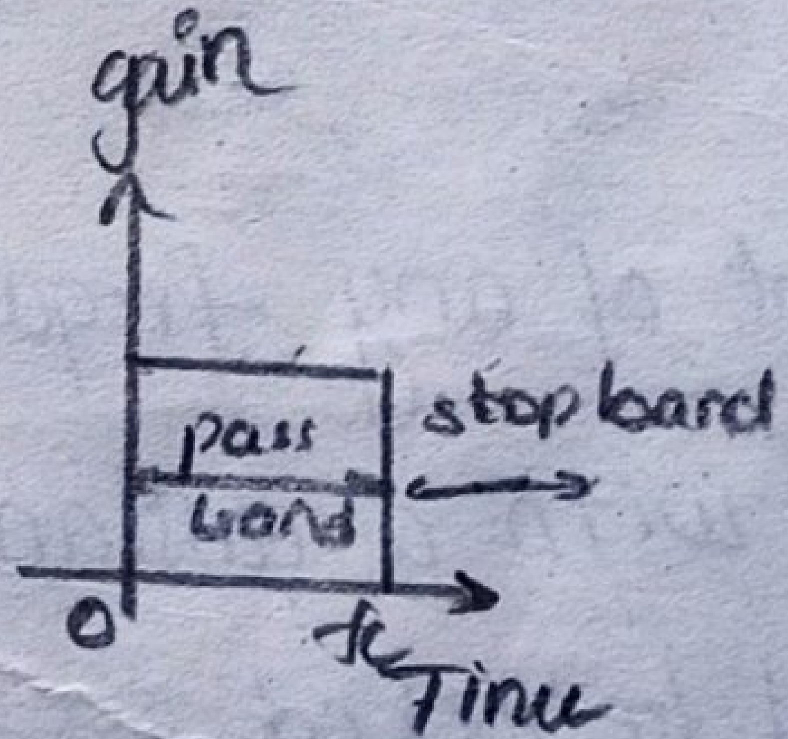
We can define that an electrical filter usually freq-uency selective network. That passes a specify band of frequencies and ~~large~~ log block signals of frequies outsid the band

classification of filters:-

(a) low pass filter (b) High pass filter (c) Band pass filter
(d) Band stop filter

Low pass filter:-





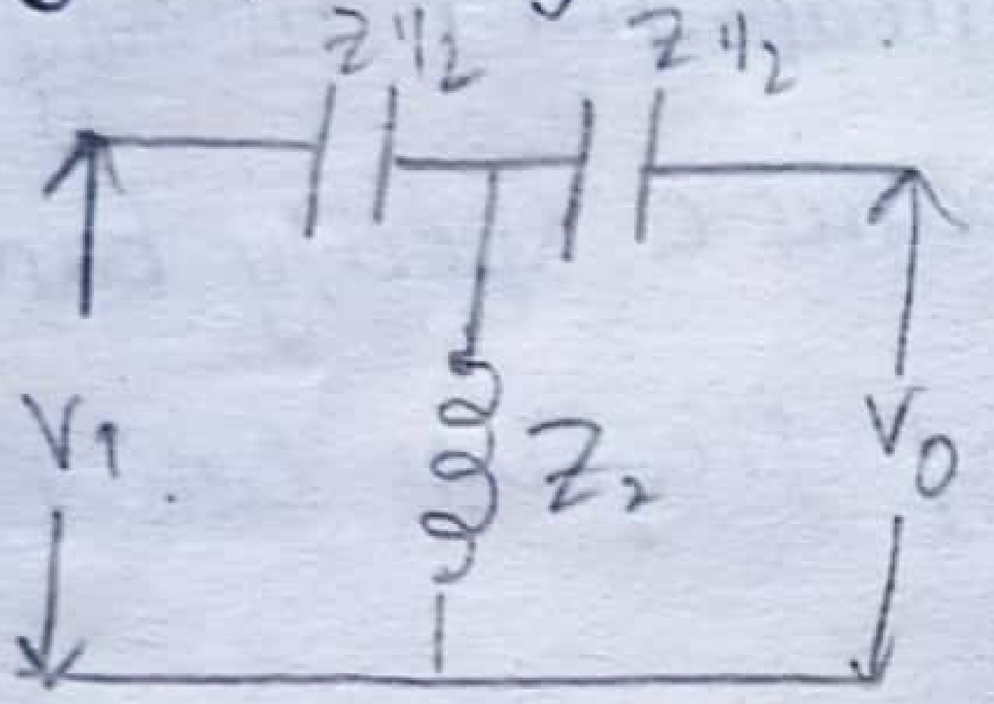
A filter that provides a constant o/p from DC upto a cutt off frequency (fc) & then ~~passes~~ no signal above the frequeny is called ideal low pass filter

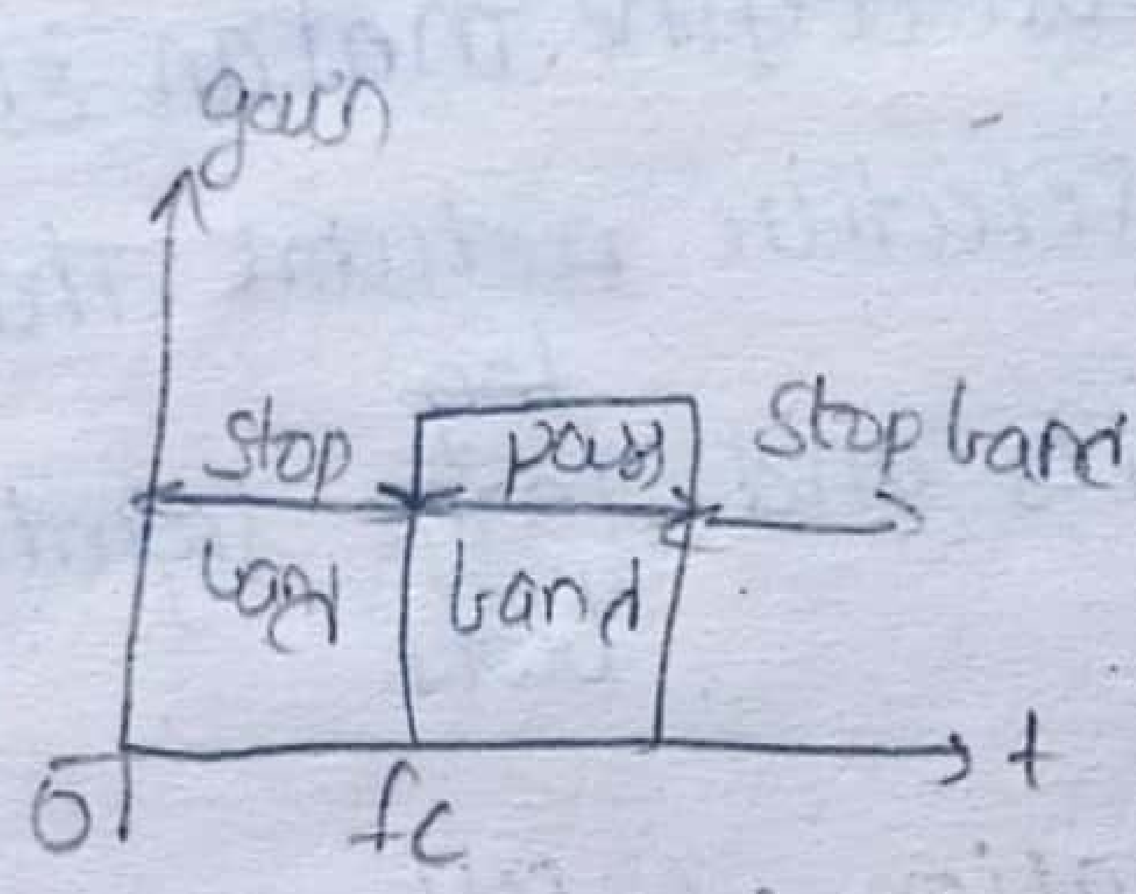
The voltage gain that is the ratio of output voltage to input voltage is constant over a frequency range from 0 to cutt of frequency (fc). Hence the output will be available from 0 to fc with constant gain

The frequency between 0 & fc are called as pass band frequency, by the frequency is above f_c are called as stop band frequency

∴ The Bandwidth of low pass frequency is (f_c)

High pass filter:-



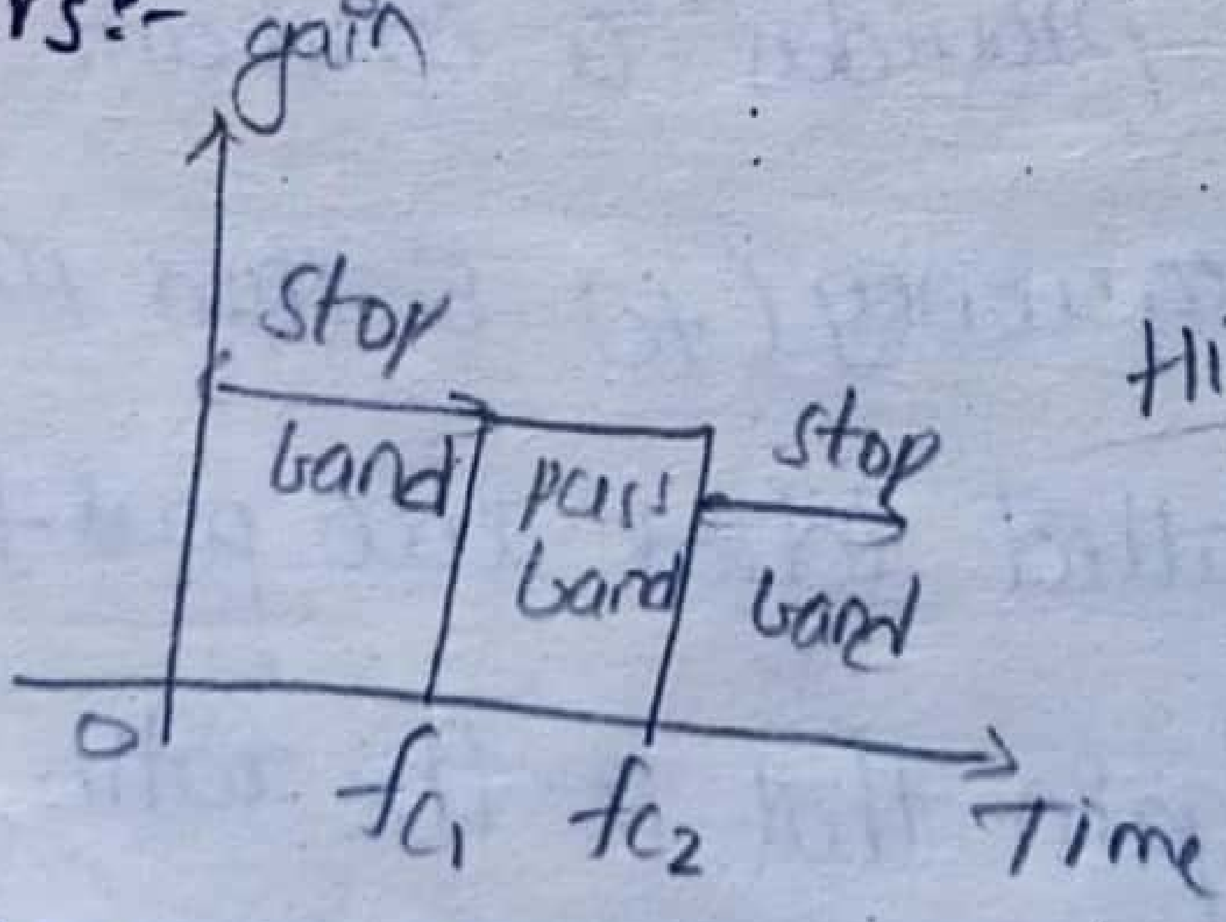


A filter that provides signal above a cut off frequency is a High pass filter

NOTE:-

Hence signal of any frequency beyond (f_c) is faithfully reproduced with a constant gain & frequencies from 0 to fc will be ~~locked~~ dropped

Band pass filters:-



High pass circuit

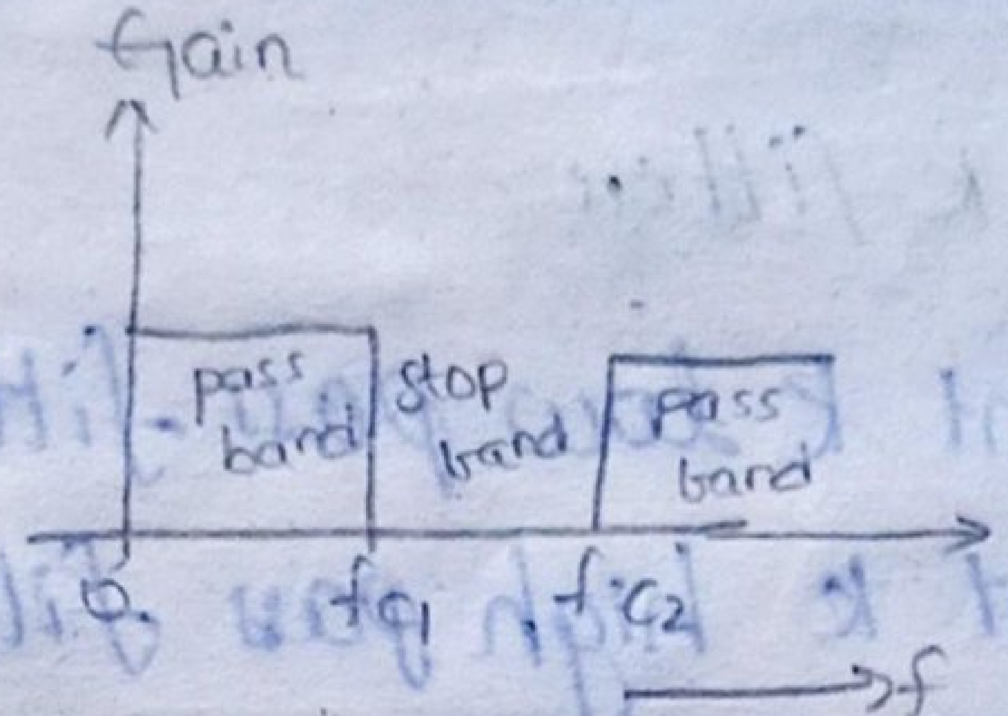
Band pass filters has a pass band b/w two cut off frequencies f_{c_2} & f_{c_1} & ~~two~~ where $f_{c_2} > f_{c_1}$

And two stop bands 0 to f_{c_1} => $0<f<f_c$ &
$f>f_{c_2}$

Band width of band pass filter is $f_{c_2}-f_{c_1}$

Where f_{c_1} & f_{c_2} are lower&higher cutt off frequencies respectively

Band stop filters:-

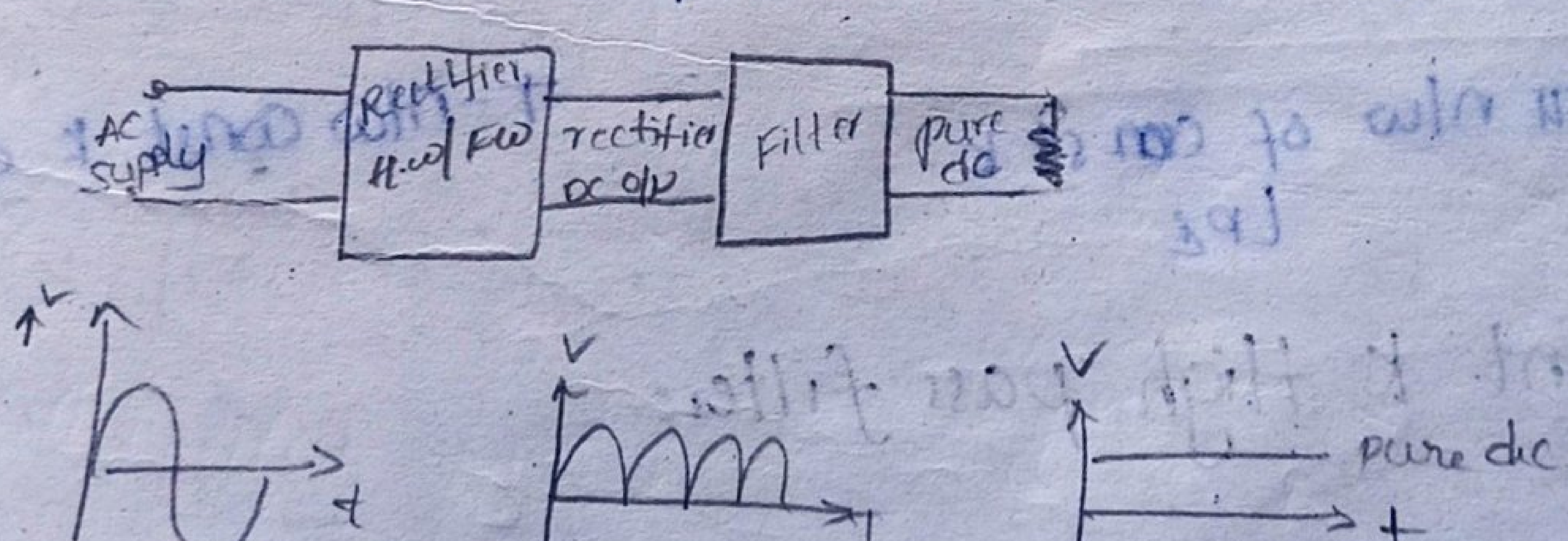


It has band stop between two cutt off frequencies f_{c_2} & f_{c_1} And two pass band $0<f<f_{c_1}$ & $f>f_{c_2}$

This is called as band Elemination or nortch filter

Need of filter circuit:-

- Output obtain by rectifier circuit is pulsating dc
- Ac component present in o/p voltage is called as ripple
- Our aim is to output should be ripple free (pure dc)



- capacitors and inductors are used for filter circuits
- Applications of filters

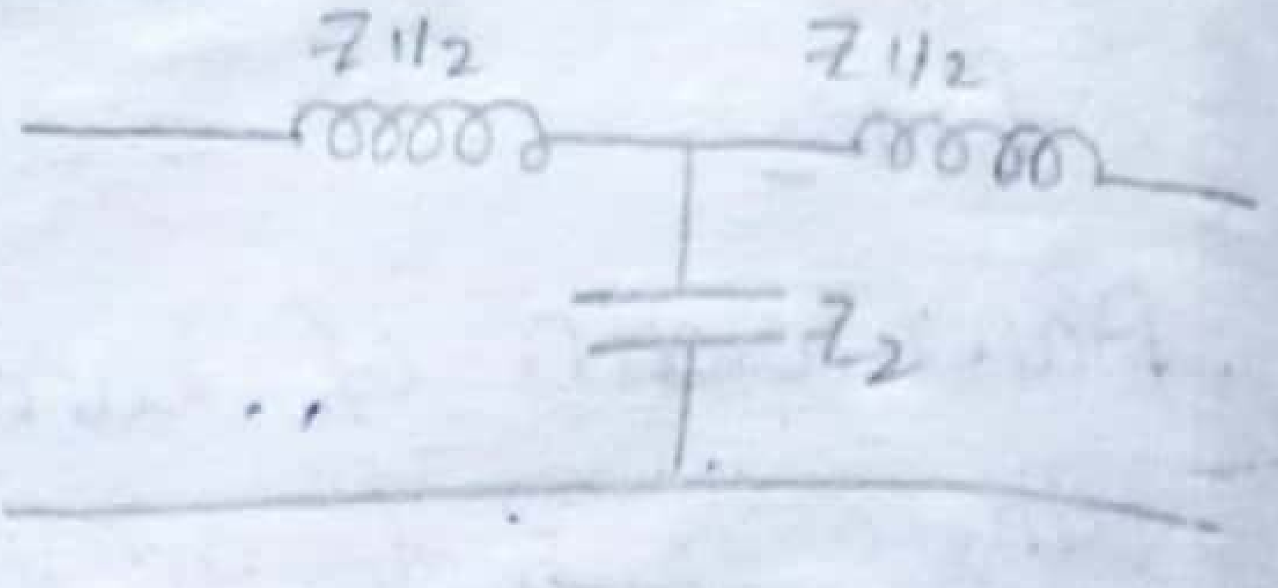
(1) Constant k filters.

(2) M derive filters.

(3) composite filters.

constant k filters:-

In this filter, the series & shunt impedences are such that



$Z_1 \times Z_2 = R_0^2 = K$ [constant]

where R_0 = Real number & independent of frequency

This is called as designed/desired impedence.

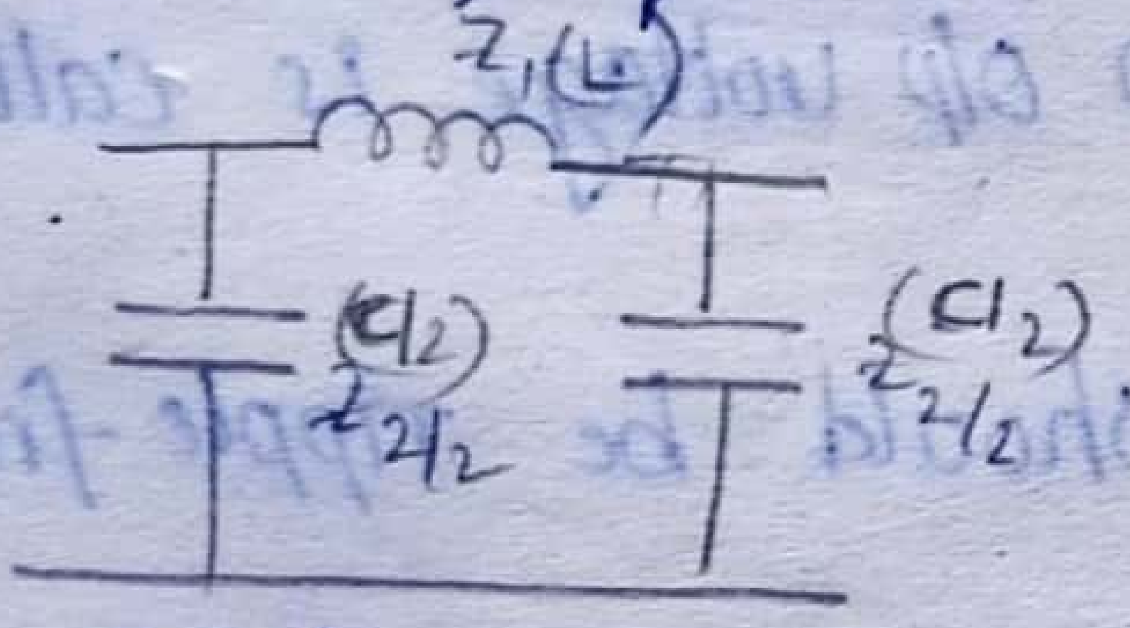
Types of K filters:-

(a) constant K low pass filters

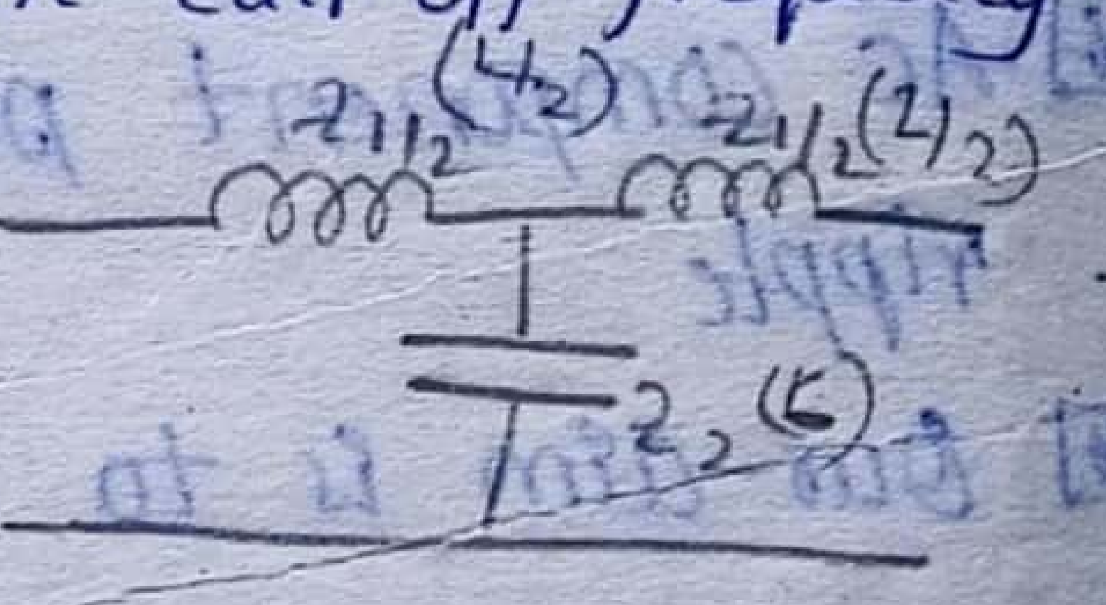
(b) constant k high pass filters

(a) constant k low pass filters:-

It is the simplest type of filter which allows the all frequencies upto the specify cutt off frequencies to pass through it and attenuates attenuation all the other frequencies above the cutt off frequency.

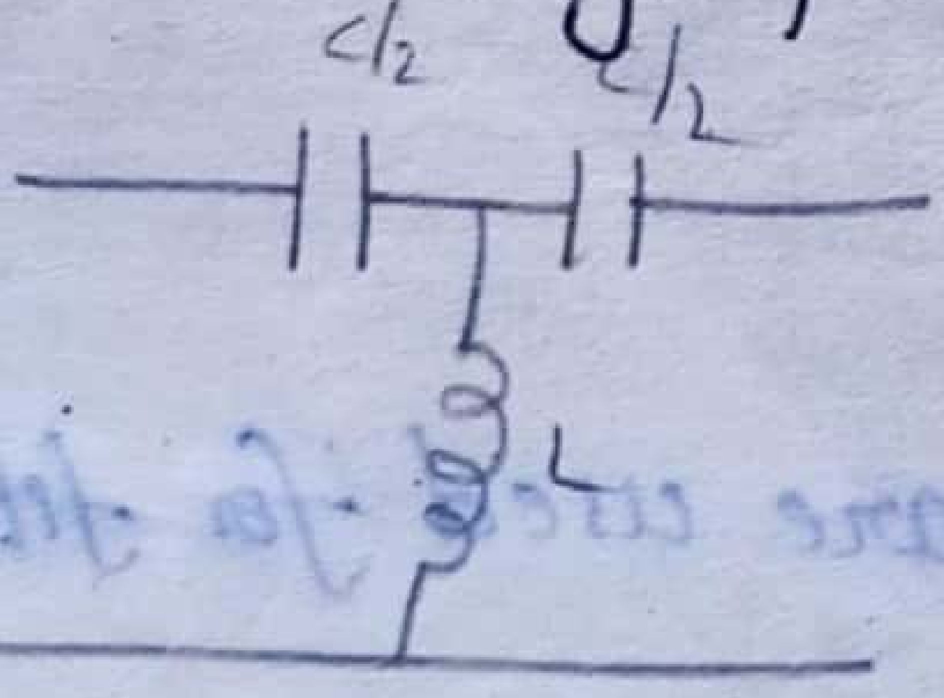


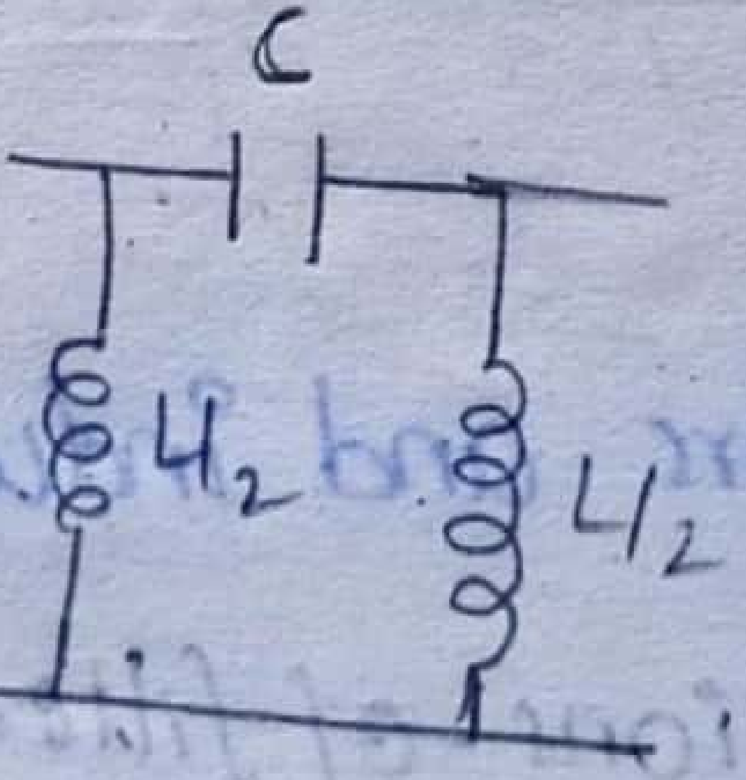
π n/w of const k LPF



T-n/w const k LPF

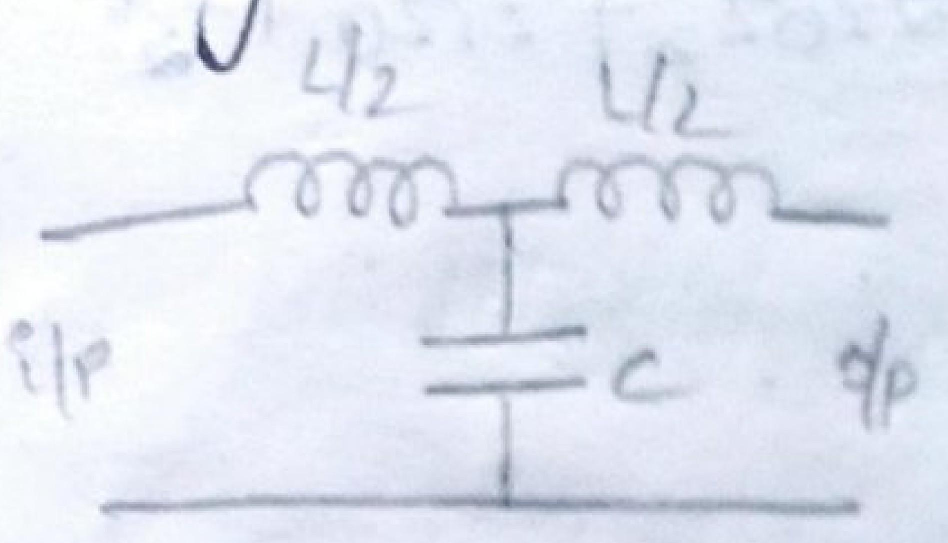
Constant k High pass filter:-



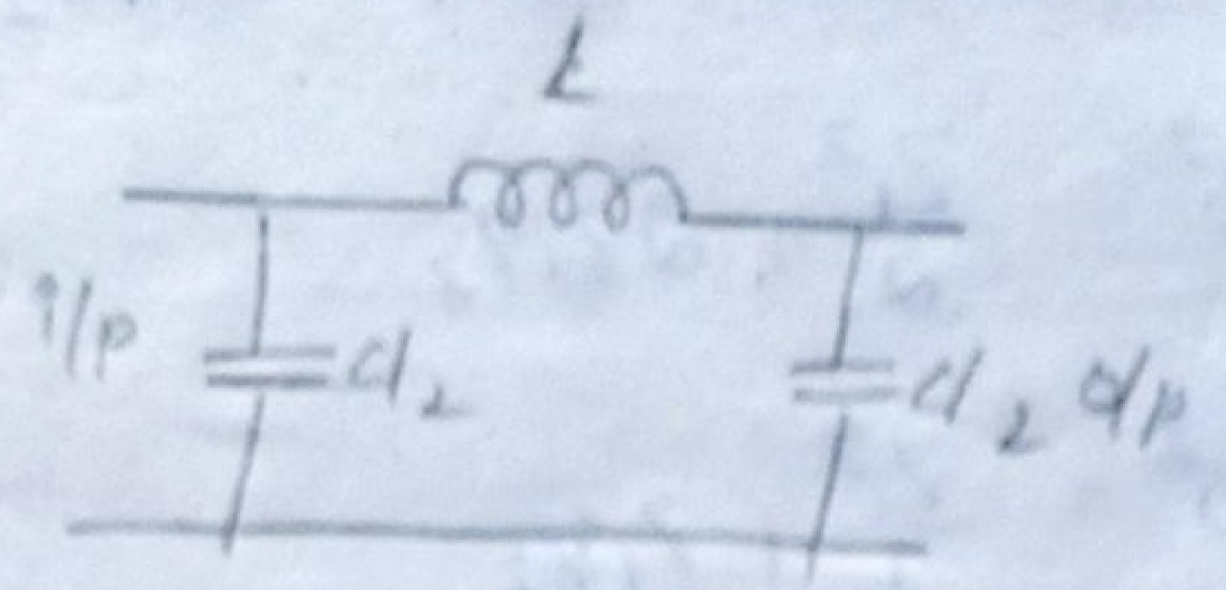


* This filter attenuates all frequency below the cutt off frequency and allow to pass all other frequencies above cut off frequency

Analysis of constant k low pass filters:



T network constant k



π network constant k

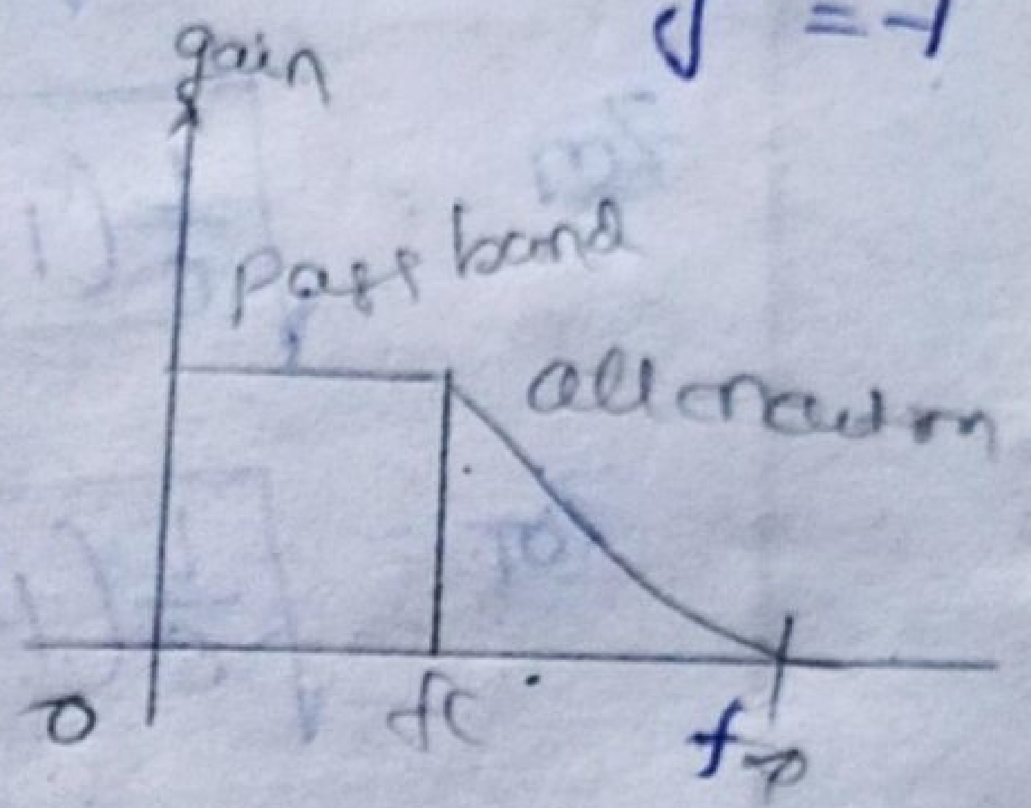
* from the figure T & π section of low pass filters
* Total series impedence $Z_1 = J\omega L \rightarrow ①$ & $Z_1 = \frac{1}{J\omega C} \rightarrow ②$
* Total shunt impedence $Z_2 = \frac{1}{J\omega C} = \frac{j}{j \times j \times \omega C}$

$= \frac{-j}{\omega C} \rightarrow ②$ $\quad i^2 = -1$, $j^2 = -1$

Multiply eq ① & eq ②

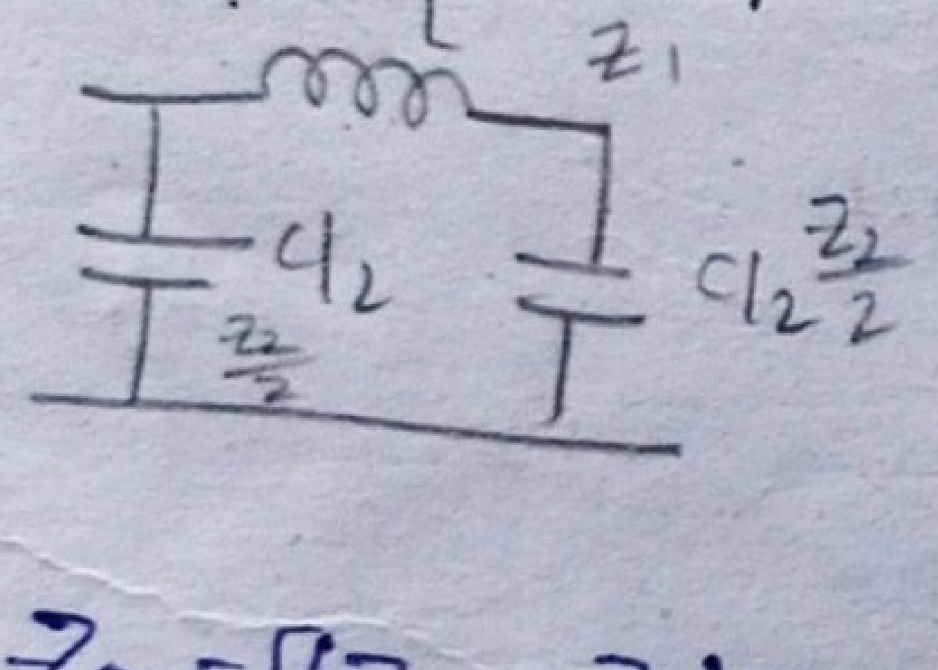
$\Rightarrow Z_1 \times Z_2 = J\omega L \times \frac{-j}{\omega C}$

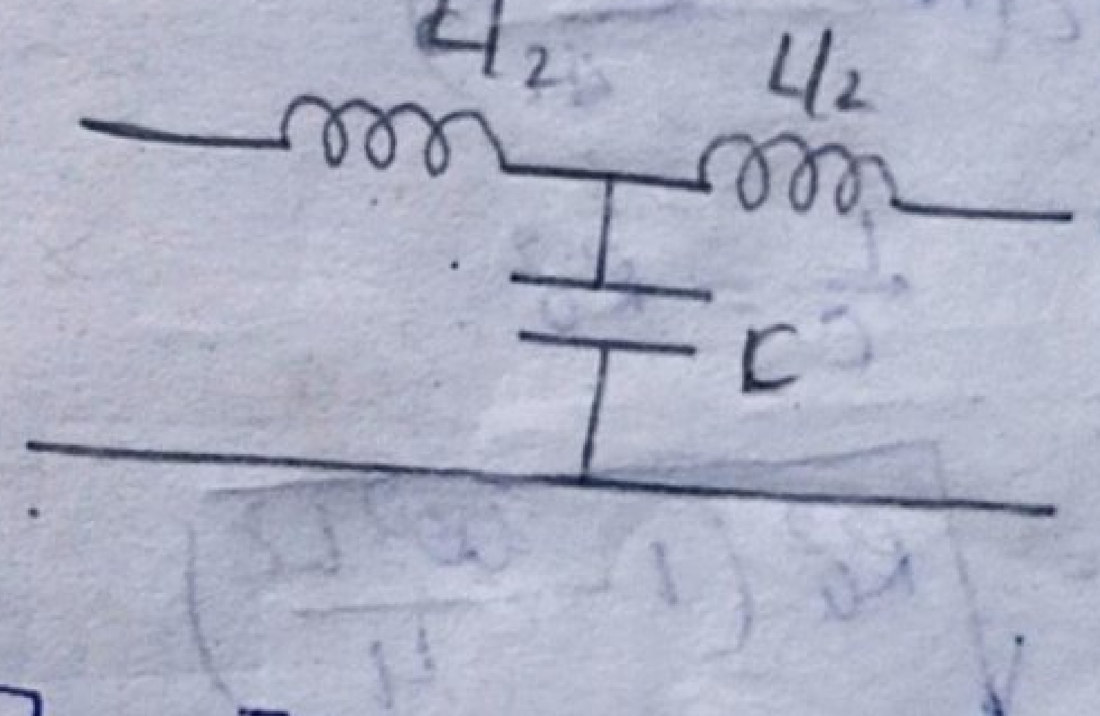
$$\boxed{Z_1 Z_2 = \frac{L}{C} = R_0^2 = K} \rightarrow ①$$



Since $\frac{L}{C}$ = Real quantity

characteristic impedence





$Z_0 = \left[\left(Z_L + \frac{Z_1}{2}\right) \| Z_2\right] + \frac{Z_1}{2}$

$Z_0 = \sqrt{\frac{Z_1 Z_2}{Z_{0\pi}}}$

$Z_0 = \frac{\left(Z_L + \frac{Z_1}{2}\right) Z_2}{Z_L + \frac{Z_1}{2} + Z_2} + \frac{Z_1}{2}$

$Z_0 = \frac{2Z_L Z_2 + Z_1 Z_2 + Z_1 Z_L + \frac{Z_1^2}{2} + Z_1 Z_2}{2Z_L + 2Z_2 + Z_1}$

$Z_0 = \frac{2Z_1 Z_2 + 2Z_L Z_2 + Z_1 Z_L + \frac{Z_1^2}{2}}{2Z_L + 2Z_2 + Z_1}$

$2Z_0 Z_L + 2Z_0 Z_2 + Z_0 Z_1 = 2Z_1 Z_2 + 2Z_L Z_2 + Z_1 Z_L + \frac{Z_1^2}{2}$

$$2Z_0^2 + 2Z_0 Z_2 + Z_0 Z_1 = 2Z_1 Z_2 + 2Z_0 Z_2 + Z_1 Z_0 + \frac{Z_1^2}{2}$$

$$2Z_0^2 = \frac{Z_1^2}{2} + 2Z_1 Z_2$$

$$Z_0^2 = \frac{Z_1^2}{4} + Z_1 Z_2$$

$$Z_0^2 = Z_1 Z_2\left(1 + \frac{Z_1}{4Z_2}\right)$$

$$Z_0 = \sqrt{Z_1 Z_2\left(1 + \frac{Z_1}{4Z_2}\right)}$$

characterstic impedence for T network Z_0

$$Z_{0T} = \sqrt{Z_1 Z_2\left(1 + \frac{Z_1}{4Z_2}\right)} \longrightarrow (4)$$

$$Z_{0T} = \sqrt{\frac{L}{C}\left(1 + \frac{Z_1}{Z_2}\right)} \quad \because \text{from eq } (3)$$

$$Z_{0T} = \sqrt{\frac{L}{C}\left(1 + \frac{j\omega L}{\frac{4 \cdot 1}{j\omega C}}\right)}$$

$$Z_{0T} = \sqrt{\frac{L}{C}\left(1 - \frac{\omega^2 LC}{4}\right)}$$

we k.T $\frac{L}{C} = R_0^2$

$$Z_{0T} = \sqrt{R_0^2\left(1 - \frac{\omega^2 LC}{4}\right)}$$

$$Z_{0T} = R_0\sqrt{\left(1 - \frac{\omega^2 LC}{4}\right)}$$

$$= R_0\sqrt{1 - \frac{\omega^2}{(4/LC)}}$$

$\omega_c^2 = \frac{4}{LC}$ Angular cut off frequency

$$= R_0\sqrt{1 - \frac{\omega^2}{\omega_c^2}}$$

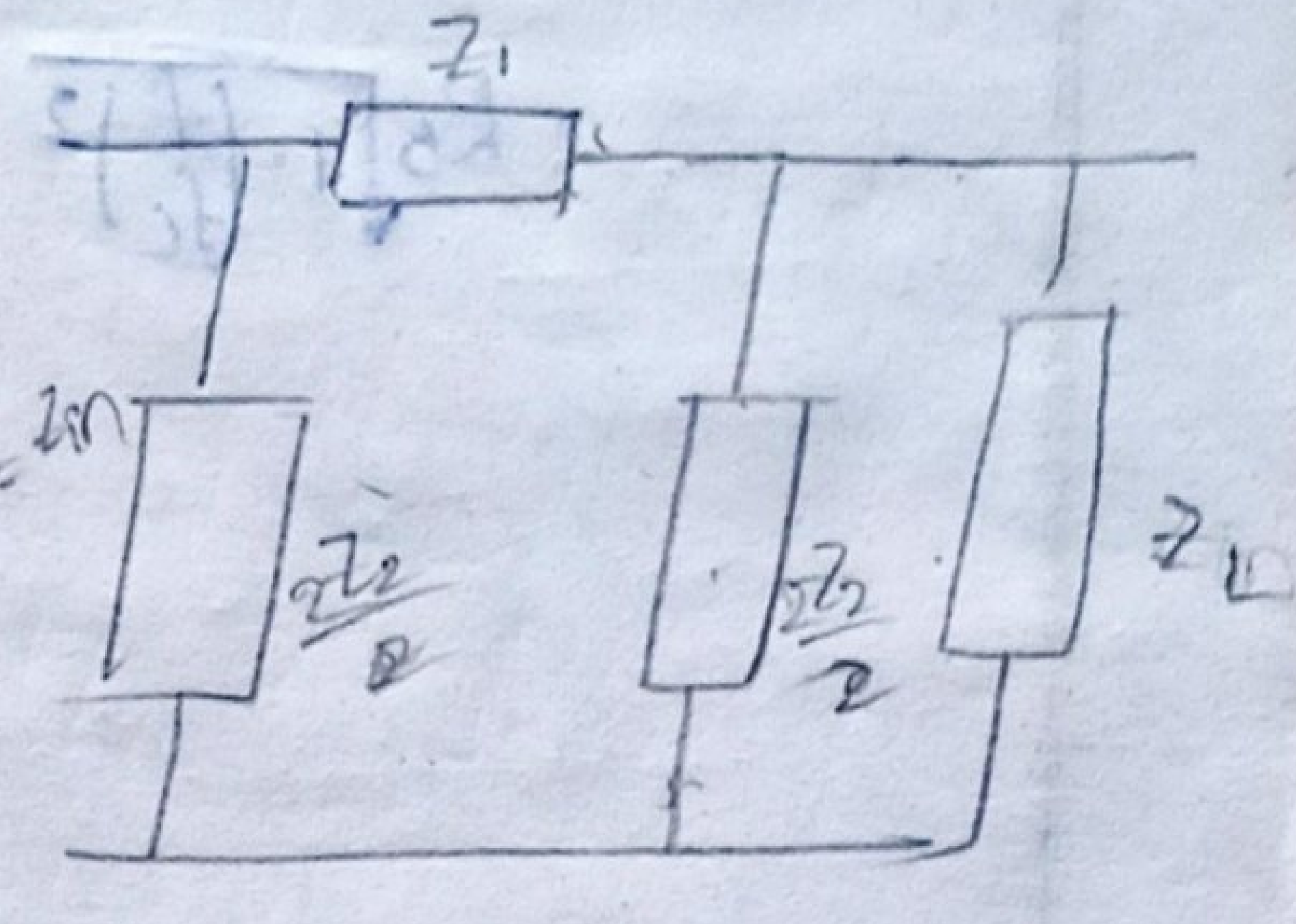
cut off frequency will be at particular condition

$$1 - \frac{\omega^2}{\frac{4}{LC}} = 0 \qquad 1 = \frac{\omega^2}{4/LC} \qquad \omega^2 = \frac{4}{LC}$$

$$\omega_c^2 = 4/LC$$

$$Z_{0T} = R_0\sqrt{1 - \frac{(2\pi f)^2}{(2\pi f_c)^2}}$$

$$= R_0\sqrt{1 - \left(\frac{f}{f_c}\right)^2}$$



$$Z_0 = \left[\left(Z_L \parallel \frac{Z_2}{2}\right) + Z_1\right] \parallel \frac{Z_2}{2}$$

$$Z_0 = \left[\frac{\left(Z_L \cdot \frac{Z_2}{2}\right)}{\left(Z_L + \frac{Z_2}{2}\right)} + Z_1\right] \parallel \frac{Z_2}{2}$$

$$Z_0 = \frac{Z_L\frac{Z_2}{2} + Z_1\left(Z_L + \frac{Z_2}{2}\right)}{\left(Z_L + \frac{Z_2}{2}\right)} \parallel \frac{Z_2}{2}$$

$$Z_0 = \left[\frac{Z_L\frac{Z_2}{2} + Z_1Z_L + \frac{Z_1Z_2}{2}}{Z_L + \frac{Z_2}{2}}\right] \parallel \frac{Z_2}{2}$$

$$Z_0 = \frac{\left(\frac{Z_L\frac{Z_2}{2} + Z_1Z_L + \frac{Z_1Z_2}{2}}{(Z_L + Z_2/2)}\right)\frac{Z_2}{2}}{\frac{Z_L\frac{Z_2}{2} + Z_LZ_1 + \frac{Z_1Z_2}{2}}{Z_L + \frac{Z_2}{2}} + \frac{Z_2}{2}}$$

$$Z_0 = \frac{\frac{Z_LZ_2^2 + Z_1Z_2Z_L + Z_1Z_2^2}{2\left(Z_L + \frac{Z_2}{2}\right)}}{\frac{2\left[\left(Z_L\frac{Z_2}{2}\right) + \left(Z_LZ_1\right) + \left(\frac{Z_1Z_2}{2}\right)\right] + Z_2\left(Z_L + \frac{Z_2}{2}\right)}{2\left(Z_L + \frac{Z_2}{2}\right)}}$$

$$Z_0 = \frac{Z_LZ_2^2 + Z_1Z_2Z_L + Z_1Z_2^2}{2Z_L\frac{Z_2}{2} + 2Z_LZ_1 + \frac{2Z_1Z_2}{2} + Z_LZ_2 + \frac{Z_2^2}{2}}$$

$$Z_0\left(\frac{Z_2^2}{2} + 2Z_LZ_2 + 2Z_LZ_1\right) = Z_LZ_2^2 + Z_1Z_2Z_L + Z_1Z_2^2$$

$$Z_0\frac{Z_2^2}{2} + 2Z_0^2Z_2 + 2Z_0^2Z_1 = Z_0Z_2^2 + Z_0Z_1Z_2 + Z_1Z_2^2$$

for T system

Value of Zeq

condition -

$$Z_{0T} = R_0\sqrt{1 - \frac{\omega^2 LC}{4}}$$

$$= R_0\sqrt{1 - \left(\frac{f}{f_c}\right)^2}$$

If $\frac{\omega^2 LC}{4}$ is less than 1, Z equivalent will be real and if $\frac{\omega^2 LC}{4}$ is greater than 1, Z equivalent will be imaginary

Hence Z_{0T} will be characteristic impedence of pass band when $\frac{\omega^2 LC}{4} < 1$ i.e when Z_{0T} is real And

Z_{0T} will characterstic impedence for stop band when $\frac{\omega^2 LC}{4} > 1$ i.e when Z_{0T} is imaginary.

for π system

$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}}$$

$$Z_1 Z_2 = R_0^2$$

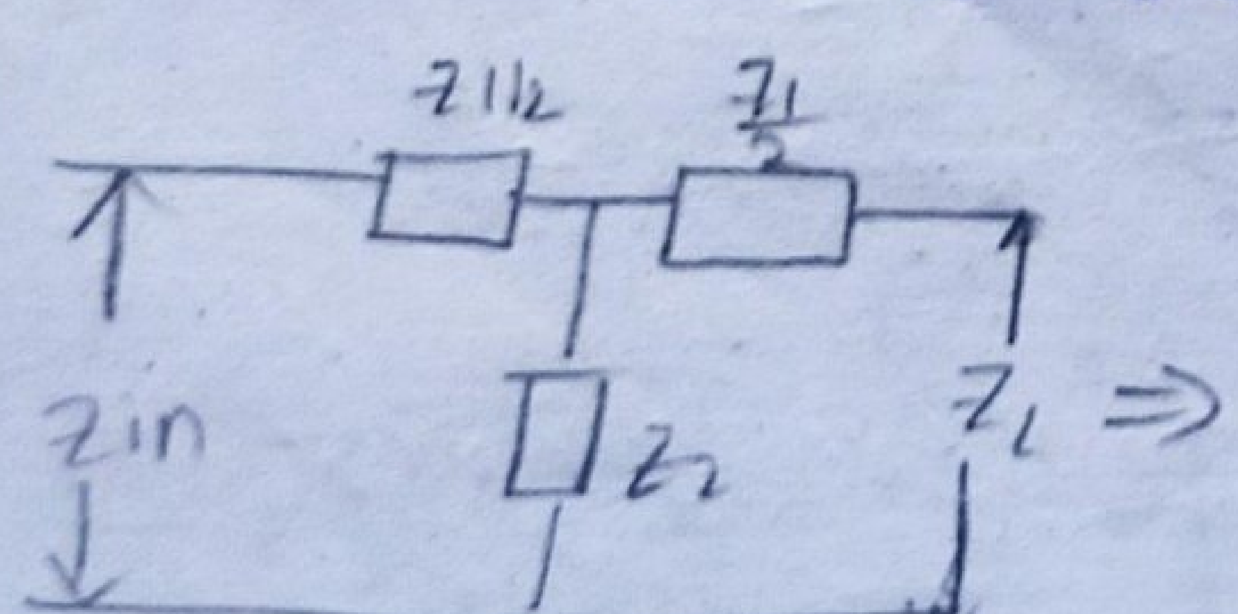
$$Z_{0T} = R_0\sqrt{1 - \left(\frac{f}{f_c}\right)^2}$$

$$Z_{0\pi} = \frac{R_0^{\cancel{2}}}{\cancel{R_0}\sqrt{1 - \left(\frac{f}{f_c}\right)^2}}$$

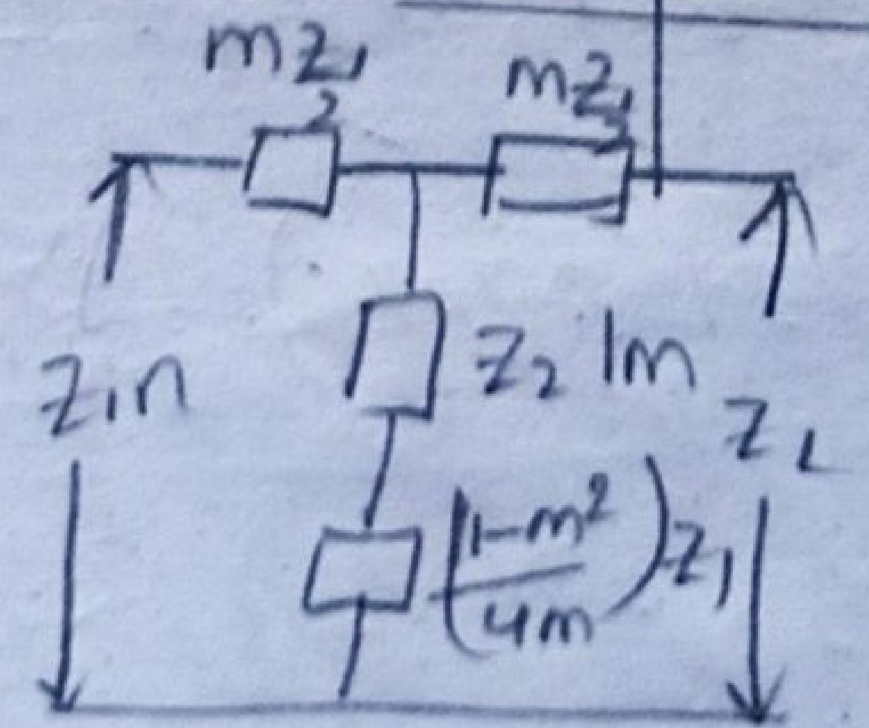
$$Z_{0\pi} = \frac{R_0}{\sqrt{1 - \left(\frac{f}{f_c}\right)^2}}$$

M-derive filter:-

* No sharp cut off between pass band & stop band
* characterstic impedence varies widely in the pass band from the derived value
*



⇒



Characterstic of M-derived filter:-

When we find out R_0 & f_c values we can easily desired m-derive filter

for T-Network.

K	m-derived filter
$\frac{Z_1}{2}$	$\frac{mZ_1}{2}$
Z_2	$\frac{Z_2}{m} + \left(\frac{1-m^2}{4m}\right)Z_1$

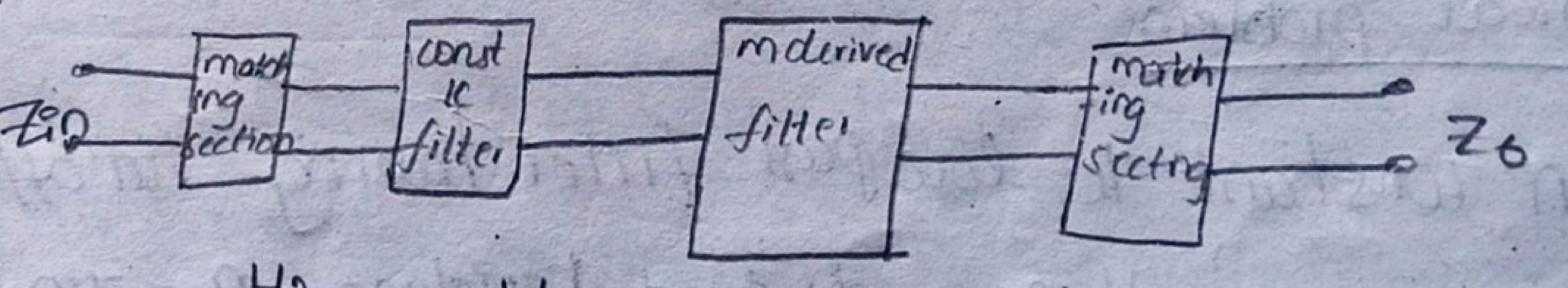
for π network

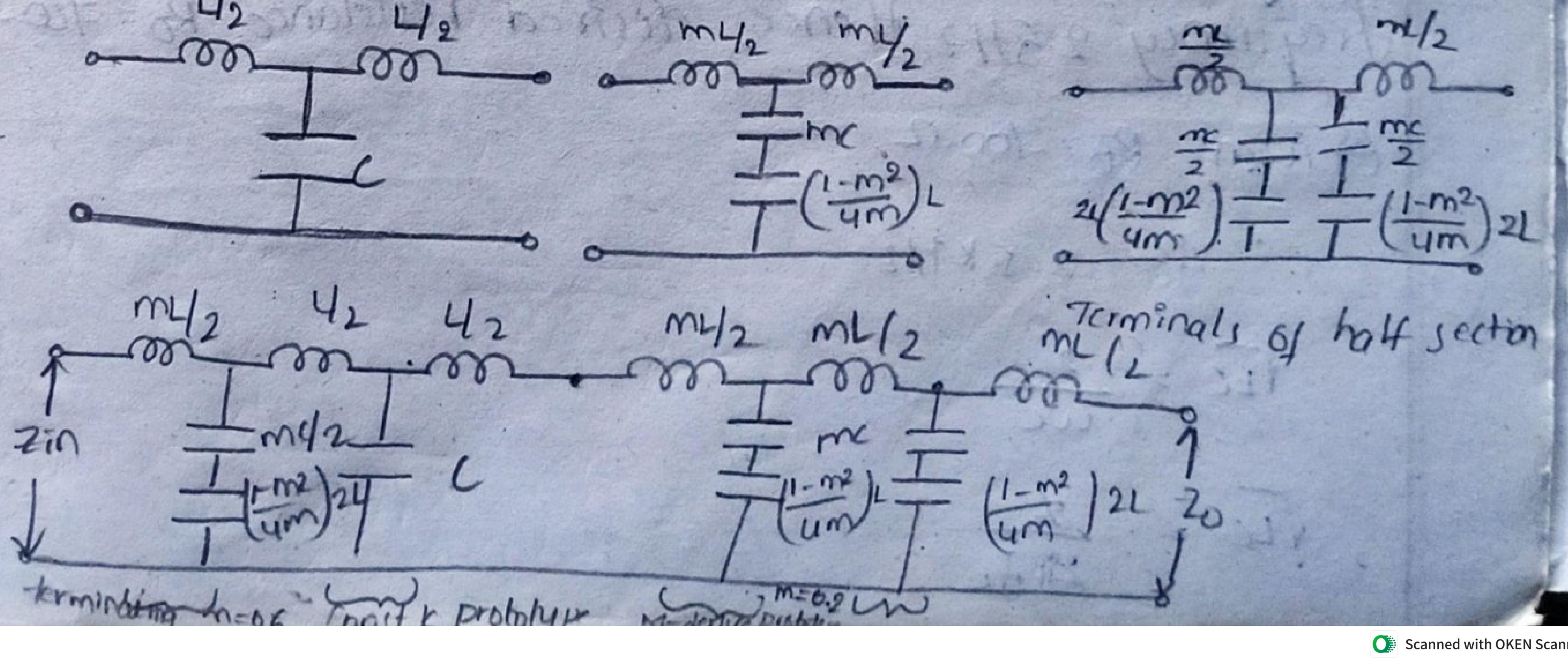
K	m-derived filter
Z_1	$mZ_1 \parallel \left(\frac{4m}{1-m^2}\right)Z_2$
$2Z_2$	$\frac{2Z_2}{m}$

Composite filters:- Const k filter + m derived filter +

Composite filter is made up of by caskading constant of k & m derived filters getting purified signal in additional with matching sections also

Note that, the purpose matching section is to provide desired impedence statistics





Terminating half Section with m=0.6
constant k prototype

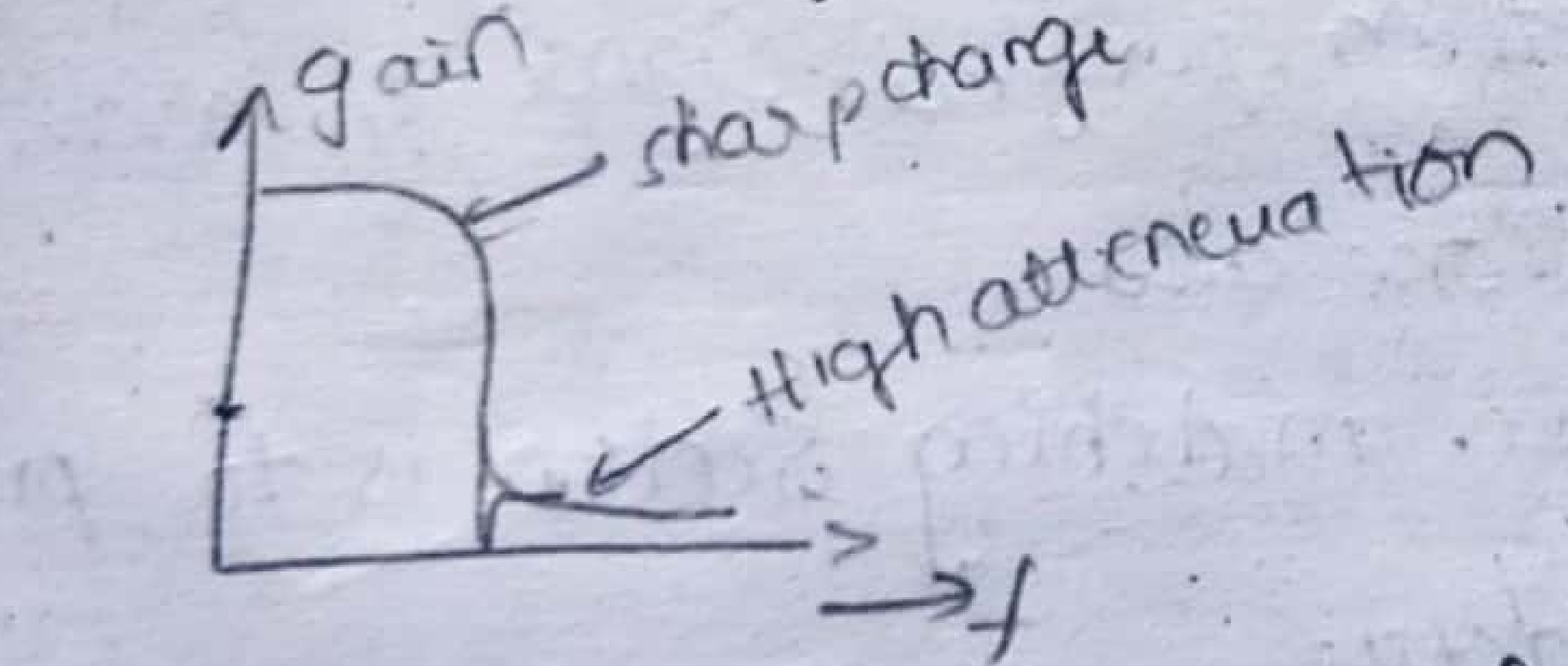
Applications of low pass filters

* They are used in Electronic & power Supply because they allow direct current but not the variations of the current & voltage
* They are also used in voice frequency circuits which passes frequency upto 3KHz
* They are use b/w transmitter & Antenna for preventing high frequency from appearing field in antenna.

* Uses of High pass filter:
* Modern digital image processing
* Sharpening the image

~~Sharpling~~

characterstics of composite filters



800 827 3231

Numerical problems

1 Design constant k low pass filter having cut'off frequency 2.5Hz Hence desired resistance $R_0 = 700\Omega$

Given $R_0 = 700\Omega$

$f_c = 2.5 KHz$

$\sqrt{LC} = \frac{2}{\omega c}$

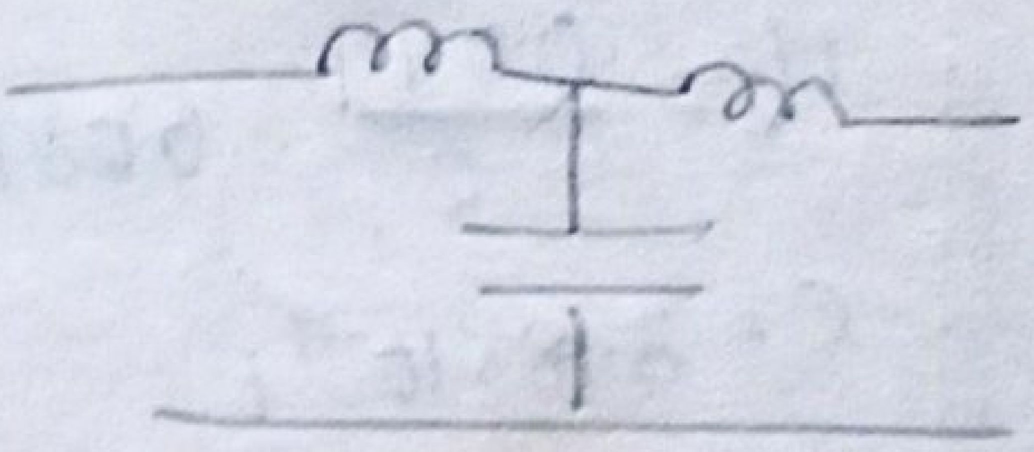
$\sqrt{LC} = \frac{\cancel{2}}{2\pi f_c}$

$R_0^2 = k^2 = \frac{L}{C}$

$R_0 = \sqrt{\frac{L}{C}} \rightarrow (1)$

$\omega_c^2 = \frac{4}{LC}$

$\omega_c = \frac{2}{\sqrt{LC}}$



$\sqrt{\frac{L}{\cancel{C}}} \times \sqrt{L\cancel{C}} = R_0 \times \frac{1}{\pi f_c}$

$\sqrt{L^2} = \frac{R_0}{\pi f_c}$

$L = \frac{R_0}{\pi f_0}$ | $C = \frac{1}{\pi f_c R_0}$

$= R_0 \times \frac{1}{\pi f_c}$

$= \frac{700}{\frac{\pi \times 2.5 \times 10^3}{2}}$

$= \frac{1400}{\pi \times 2.5 \times 10^3}$

$= \frac{14}{\pi\, 2.5 \times 10} = \frac{14}{\pi (25)}$

$= 0.1782$

② Design a low pass composite filter to operate with a desired impedence $500\,\Omega$, $m = 0.2$ & cut frequency $= 2000\,Hz$

Step1:- ~~da~~ L, C from characteristic impedence

Step2:- Draw const k-type-filter

Step3:- Draw the Modified derived filter with given value 0.2

Step4:- Modifying filter to half sections with m = 0.6

Step5:

$R_0 = 500\,\Omega$

$m = 0.2$

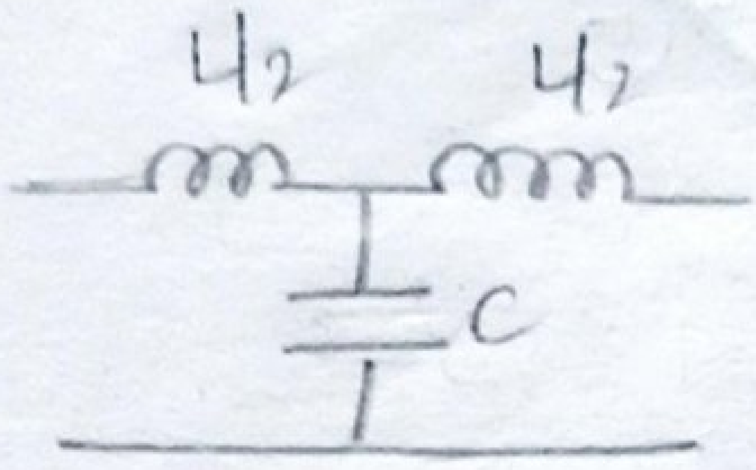
$f_c = 200\,Hz$

$L = \frac{R_0}{\pi f_c}$

$L = \frac{R_0}{\pi f_c} = \frac{500}{\pi \times 2000} = 0.0794$

$C = \frac{1}{R_0 \pi f_c} = \frac{1}{500 \times \pi \times 2000} = 3.18 \times 10^{-7} F$

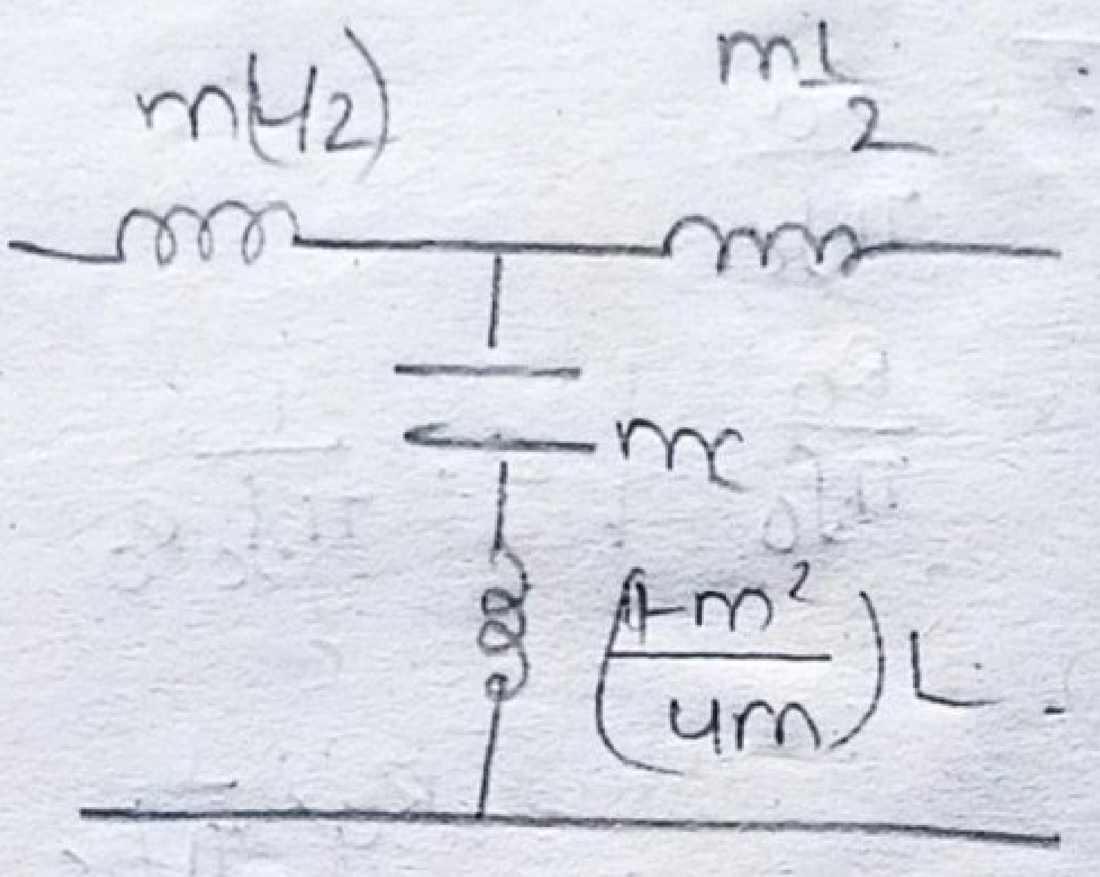
k type-filter



$L/2 = \frac{0.079}{2} = 0.0395 Hz$

$C = 3.18 \times 10^{-7} F$

M-derived filter



$mL/2 = 0.2 \times 0.0395 Hz$

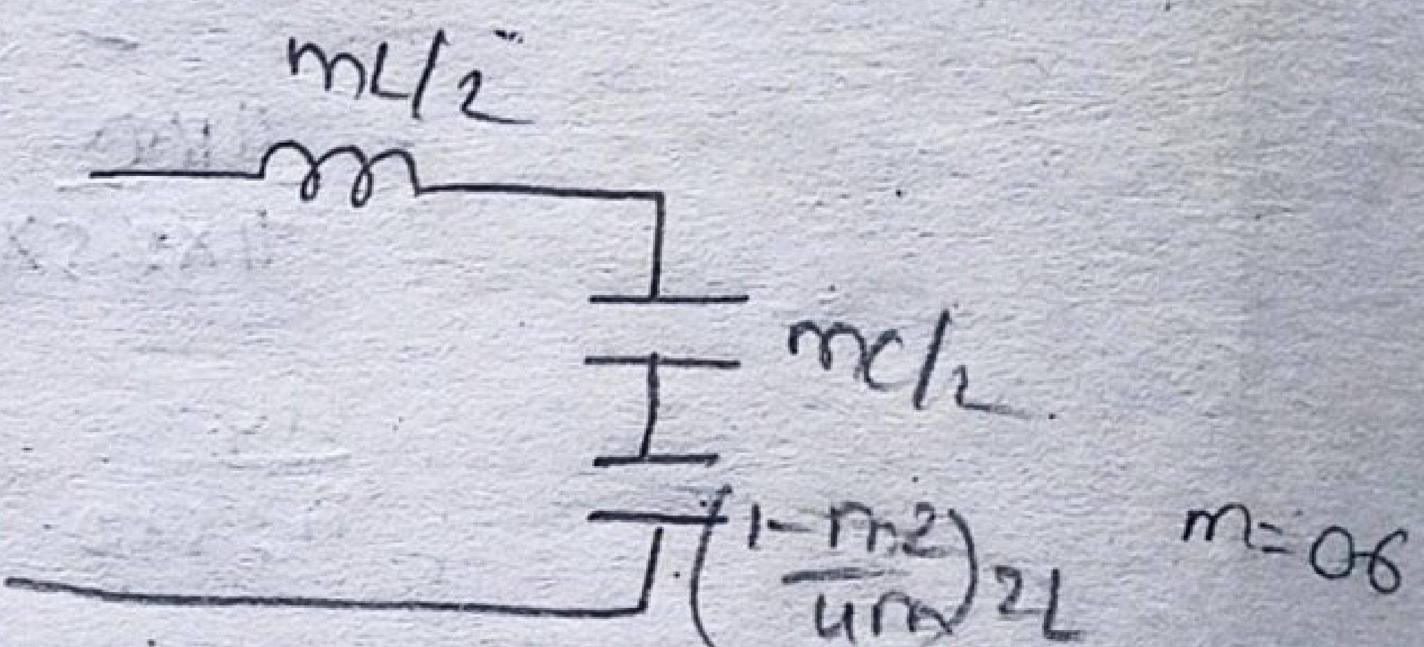
$= 7.9 mH$

$mc = 0.2 \times 3.18 \times 10^{-7}$

$= 6.36 \times 10^{-8} F$

$\left(\frac{1-m^2}{4m}\right) L = 0.095 H$

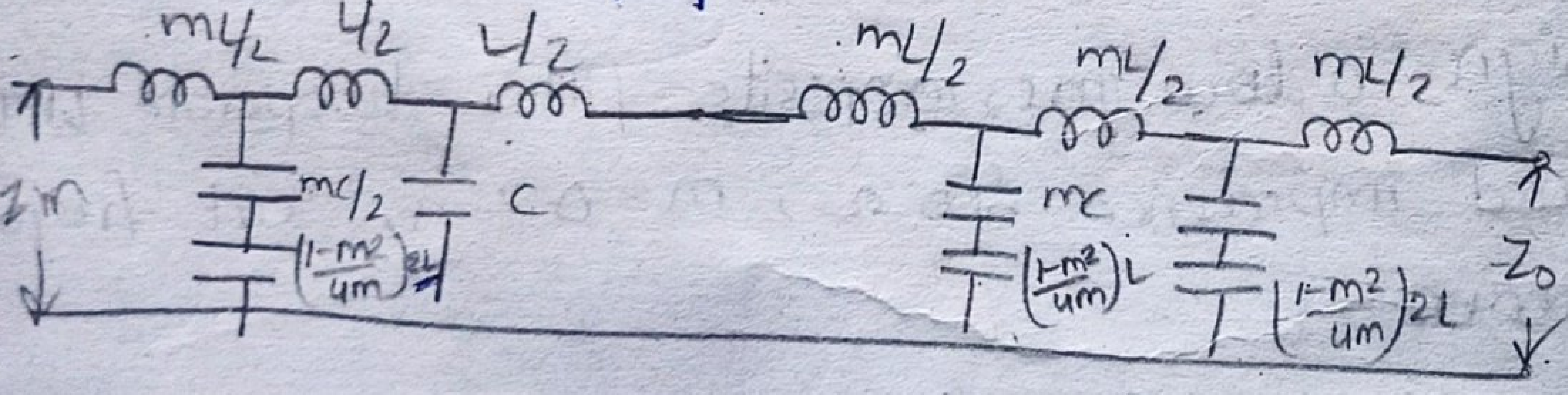
Half terminating section



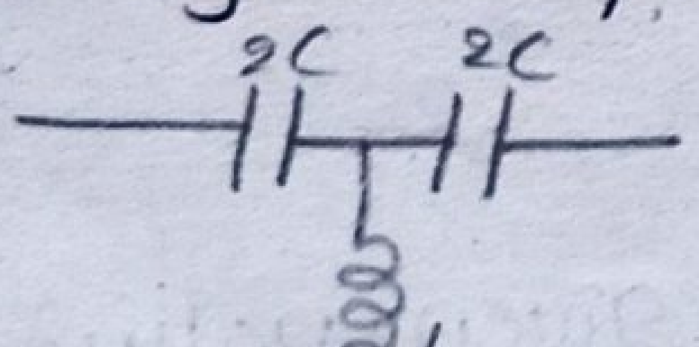
$mc/2 = 3.18 \times 10^{-8} F$

$m = 0.6$

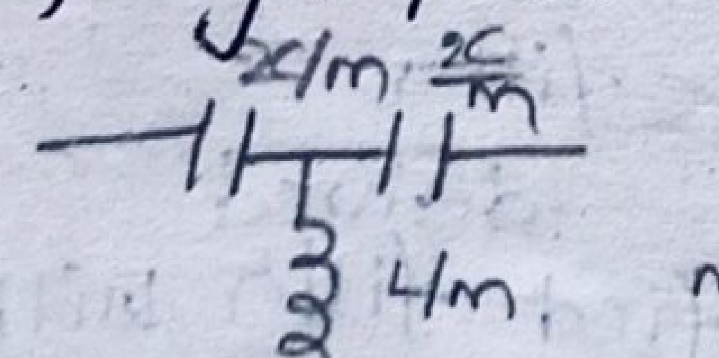
$\left(\frac{1-m^2}{4m}\right) 2L = 0.0424$



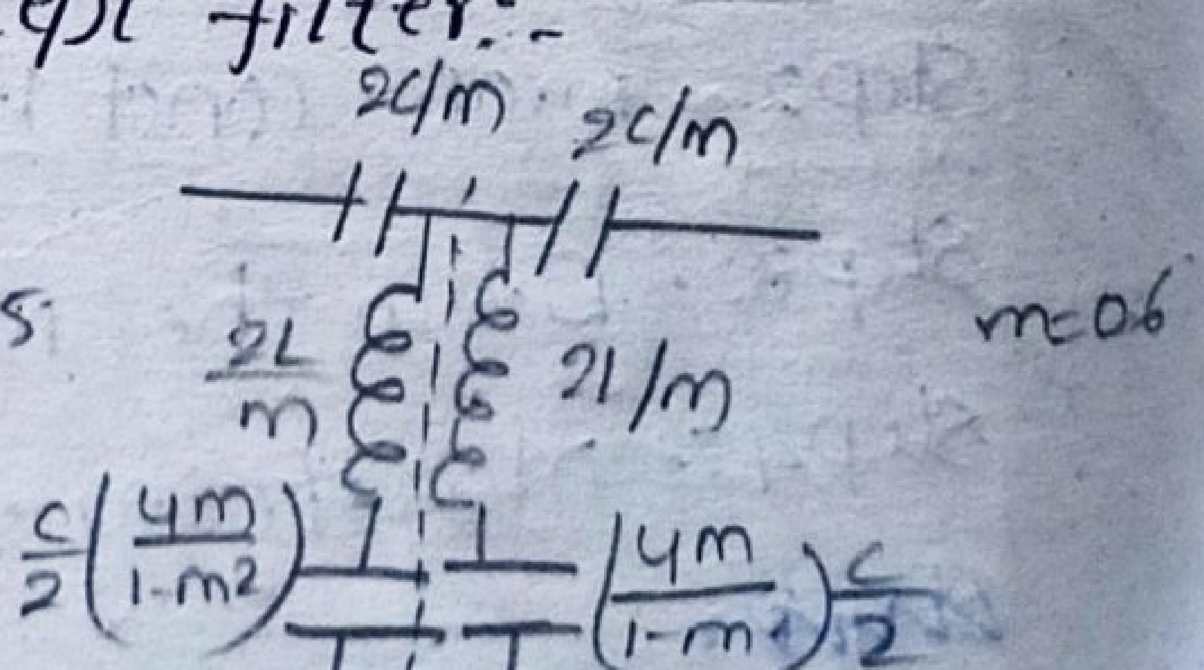
Design concept of high pass concept filter:-



const k filter



M-derived high pass



terminating half section

(11) M-derive section with m=0.4, terminating half section with m=0.6 design a complete high pass filter composit filter having the inductance of L= 40mH, C=0.1 μF

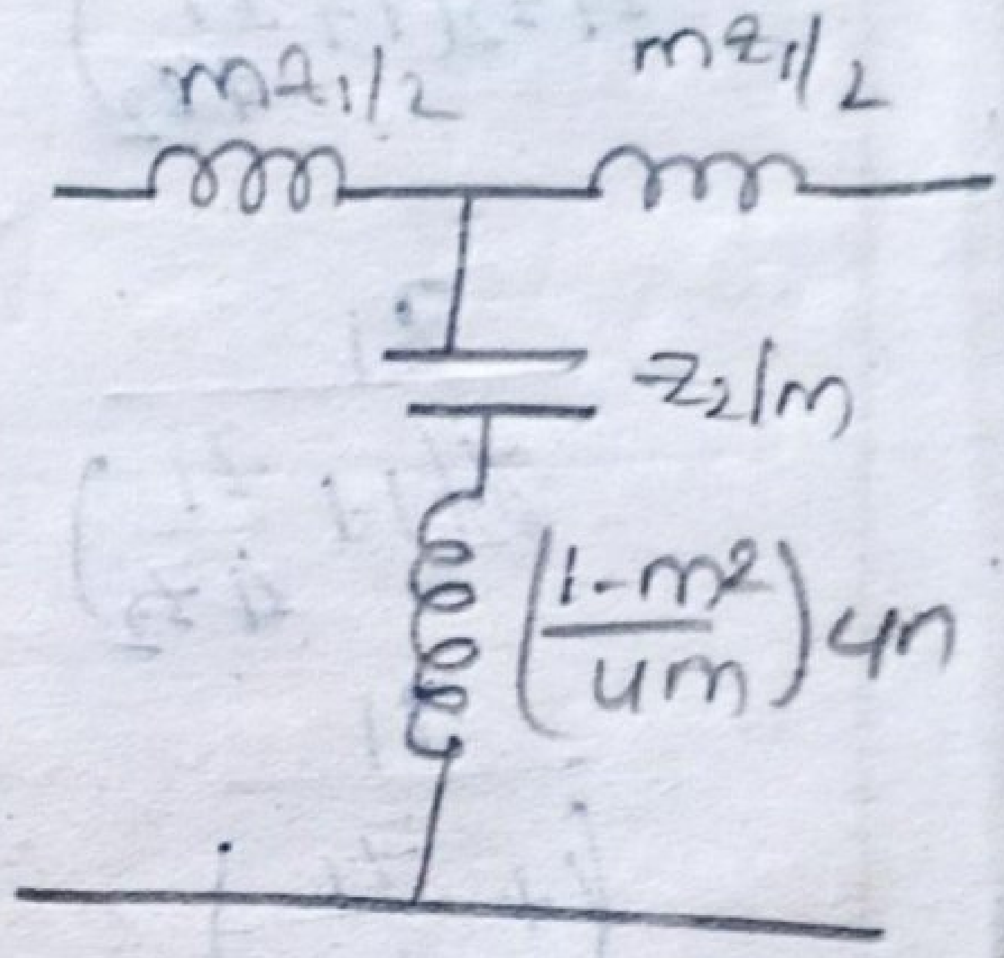
$$Z_{0T} = \sqrt{Z_1 Z_2\left(1+\frac{Z_1}{4Z_2}\right)} \rightarrow (1)$$

$$Z_{0T}' = \sqrt{mZ_1 Z_2'\left(1+\frac{mZ_1}{4Z_2'}\right)} \rightarrow (2)$$

eq (1) = eq (2)

$$Z_{0T} = Z_{0T}'$$

$$\sqrt{Z_1 Z_2\left(1+\frac{Z_1}{4Z_2}\right)} = \sqrt{mZ_1 Z_2'\left(1+\frac{mZ_1}{4Z_2'}\right)}$$



$$Z_2\left(1+\frac{Z_1}{4Z_2}\right) = mZ_2'\left(1+\frac{mZ_1}{4Z_2'}\right)$$

$$Z_2 + \frac{Z_1}{4} = mZ_2' + \frac{m^2 Z_1}{4}$$

$$\frac{m^2 Z_1}{4} - \frac{Z_1}{4} = Z_2 - mZ_2'$$

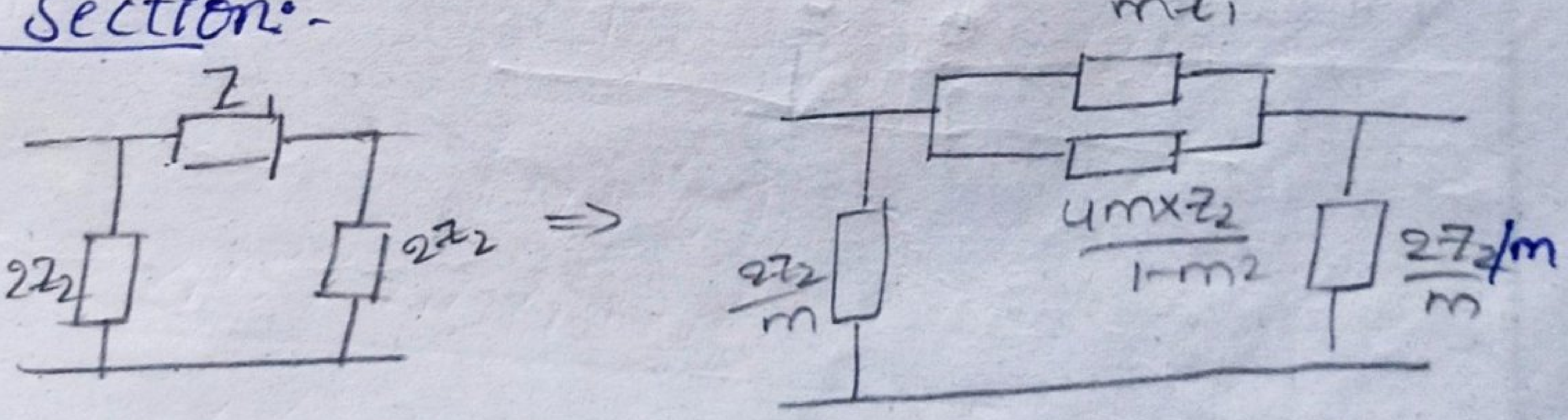
$$\frac{Z_1}{4}(m^2-1) = Z_2 - mZ_2'$$

$$-Z_2 + mZ_2' = (1-m^2)\frac{Z_1}{4}$$

$$mZ_2' = Z_2 + (1-m^2)\frac{Z_1}{4}$$

$$Z_2' = \frac{Z_2}{m} + \left(\frac{1-m^2}{4m}\right)Z_1$$

For π Section:-



$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} = \frac{Z_1 Z_2}{\sqrt{Z_1 Z_2\left(1+\frac{Z_1}{4Z_2}\right)}} \rightarrow (1)$$

$$Z_{0\pi}' = \frac{Z_1' Z_2 m}{Z_1' m Z_2\left(1+\frac{Z_1'}{4mZ_2}\right)} \rightarrow (2)$$

eq① = eq②

$$\frac{z_1 z_2}{\sqrt{z_1 z_2\left(1+\frac{z_1}{4z_2}\right)}} = \frac{z_1' m z_2}{\sqrt{z_1' m z_2\left(1+\frac{z_1'}{4z_2 m}\right)}}$$

$$\frac{z_1^2}{z_1 z_2\left(1+\frac{z_1}{4z_2}\right)} = \frac{z_1'^2 m^2}{\cancel{z_1'} m z_2\left(1+\frac{z_1'}{4z_2 m}\right)}$$

$$\frac{z_1}{\cancel{z_2}\left(1+\frac{z_1}{4z_2}\right)} = \frac{m z_1'}{\cancel{z_2}\left(1+\frac{z_1'}{4z_2 m}\right)}$$

$$\frac{z_1}{\left(1+\frac{z_1}{4z_2}\right)} = \frac{m z_1'}{\left(1+\frac{z_1'}{4z_2 m}\right)}$$

$$z_1\left(1+\frac{z_1'}{4z_2 m}\right) = m z_1'\left(1+\frac{z_1}{4z_2}\right)$$

$$\frac{z_1(4z_2 m + z_1')}{4\cancel{z_2} m} = \frac{m z_1'(4z_2 + z_1)}{4\cancel{z_2}}$$